MULTIPLYING AND DIVIDING FRACTIONS AND MIXED NUMBERS

BIG IDEAS:

- A common denominator is **not** required for multiplying or dividing fractions
- To **multiply** fractions, multiply the numerators and then multiply the denominators
- To **divide** fractions, **multiply** the first fraction by the **reciprocal** (flip the positions of the numerator and denominator) of the second

LEARNING GOALS AND SKILL DEVELOPMENT:
You know you have met the goals for this lesson when you can:

EMERGING

LEARNING GOALS	ANCHOR QUESTIONS
Multiply whole numbers by fractions	2
Divide whole numbers by fractions	9
Divide two fractions	9

SKILL BUILDING QUESTIONS			
1	2	3	4
9			

EVOLVING

LEARNING GOALS	ANCHOR QUESTIONS
Multiply two fractions, including both positive and negative fractions	11
Divide fractions and whole numbers, including positive and negative values	12
Solve problems involving the multiplication and division of fractions and whole numbers	13

SKILL BUILDING QUESTIONS			
5	6	7	8
10	11	12	13

EXTENDING

LEARNING GOALS	ANCHOR QUESTIONS
Multiply fractions and mixed numbers, including both positive and negative values	14
Divide fractions and mixed numbers, including both positive and negative values	17
Solve problems involving the multiplication and division of fractions and whole numbers with connections to other topics	16, 18, 23
Apply order of operations (BEDMAS) rules to solve multi-step problems involving fractions and whole numbers	22

SKILL BUILDING QUESTIONS			
14	15	16	17
18	19	20	21
22	23		

BUILD YOUR SKILLS

1. Explain why $3\times\frac{1}{2}$ is equivalent to $\frac{3}{2}$.

2. Express each of the following multiplications as a repeated addition and evaluate. Express your final answer as a fraction in lowest terms.

a) $2\times\frac{1}{3}$ b) $3\times\frac{1}{2}$ c) $5\left(\frac{2}{3}\right)$ d) $\left(\frac{7}{4}\right)(5)$ e) $6\left(\frac{1}{4}\right)$ f) $(9)\left(\frac{5}{12}\right)$

3. Sonja has a box of quarter-inch screws. For a current project, she estimates that she'll need screws that are three times that length. What is the length of the screws she requires?

4. Evaluate. Express each answer as a fraction in lowest terms.

a) 3 groups of $\frac{1}{5}$ b) 5 times $\frac{1}{8}$ c) $2\times\frac{1}{9}$ d) $8\left(\frac{3}{11}\right)$ e) $(7)\left(\frac{5}{16}\right)$ f) $\left(\frac{3}{4}\right)(3)$

5. Use a visual representation to show why one third of one quarter is equal to one twelfth.

6. Determine the value of each of the following.

a) Half of $\frac{1}{2}$ b) $\frac{1}{2}$ of $\frac{1}{3}$ c) $\frac{1}{2}$ of $\frac{3}{4}$ d) $\frac{1}{3}$ of $\frac{1}{2}$ e) $\frac{1}{3}$ of $-\frac{2}{5}$ f) $\frac{2}{3}$ of $\frac{4}{5}$

7. Multiply. Express each answer as a fraction in simplest (reduced) form.

a) $\frac{1}{2}\times\frac{1}{4}$ b) $\frac{1}{3}\times\frac{5}{6}$ c) $\left(\frac{2}{3}\right)\left(\frac{4}{3}\right)$ d) $\frac{5}{9}\left(\frac{7}{3}\right)$ e) $\frac{5}{8}\times\frac{2}{3}$ f) $\left(\frac{1}{5}\right)\left(-\frac{3}{4}\right)$

8. Explain why $3\div\frac{1}{4}$ is equivalent to 3×4.

9. Divide. Express each answer as a fraction in simplest (reduced) form.

a) $5\div\frac{1}{2}$ b) $4\div\frac{1}{3}$ c) $\frac{1}{3}\div\frac{1}{2}$ d) $\frac{7}{8}\div\frac{1}{3}$ e) $\frac{9}{4}\div\frac{2}{5}$ f) $-\frac{5}{6}\div\frac{4}{3}$

10. At a school talent show, one half of the acts were musical. Three quarters of the musical acts were solo performances. What fraction of the talent show consisted of solo musical performances?

11. Multiply. Express each answer as a fraction in lowest terms.

a) $7\times\frac{5}{8}$ b) $\frac{4}{9}\times\frac{5}{9}$ c) $\frac{1}{4}\left(\frac{7}{6}\right)$ d) $4\left(-\frac{3}{8}\right)$ e) $-\frac{9}{4}\left(\frac{5}{6}\right)$ f) $\left(-\frac{7}{10}\right)\left(-\frac{5}{3}\right)$

12. Divide. Express each answer as a fraction in lowest terms.

a) $\frac{2}{3}\div\frac{1}{2}$ b) $\frac{9}{5}\div\frac{3}{4}$ c) $20\div\frac{8}{3}$ d) $\frac{6}{13}\div 4$ e) $-\frac{7}{16}\div\frac{3}{2}$ f) $\frac{8}{9}\div(-4)$

13. Omar uses an adapter to change the size of the plug on his headphones cable. The original plug has a diameter of $\frac{1}{8}$" and the adapter converts this diameter to $\frac{1}{4}$".

a) Does the adapter increase or decrease the diameter of the original plug?

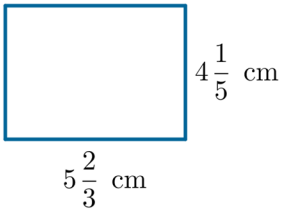
b) How many times larger is the diameter of the larger plug than that of the smaller plug?

c) By how many inches is the diameter of the larger plug greater than that of the smaller plug?

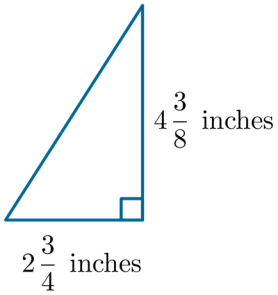
14. Multiply. Express each answer as a mixed number, where applicable.

a) $3\times 2\frac{6}{7}$ b) $4\frac{3}{5}\times 2\frac{1}{3}$ c) $\left(-7\frac{3}{4}\right)(-2)$ d) $\frac{8}{5}\left(-9\frac{10}{11}\right)$ e) $-3\frac{5}{6}\times 6$

15. Determine the area and perimeter of the rectangle on the right.



16. Determine the area of the triangle on the right.



17. Divide. Express each answer as a mixed number, where applicable.

a) $4\frac{2}{5} \div 2$ b) $3\frac{1}{2} \div 2\frac{1}{4}$ c) $-4\frac{2}{3} \div 3\frac{4}{5}$ d) $-6 \div \left(-4\frac{7}{8}\right)$ e) $6\frac{7}{10} \div \frac{2}{3}$

18. Holly took $1\frac{4}{5}$ hours to mow her lawn.

a) It took Raymond twice as long to mow his lawn. Express this time as a mixed number.

b) Jerika mowed her lawn in half the time it took Holly. Express this time as a mixed number.

c) Manny mowed his lawn in 1 hour and 24 minutes. Express this time in hours as a mixed number in simplest form.

d) Express the time it took Holly to mow her lawn as a combination of hours and minutes.

19. Show how $\frac{1257}{370} \times \frac{370}{2411}$ can be quickly evaluated without using large numbers.

20. Use the idea in question #19 to evaluate the following.

a) $\left(\frac{7}{3}\right)\left(\frac{3}{20}\right)$ b) $\left(\frac{16}{19}\right)\left(-\frac{15}{16}\right)$ c) $\frac{10}{11} \div \frac{13}{11}$ d) $\frac{57}{16} \times \frac{8}{121}$ e) $\left(\frac{27}{20}\right)\left(\frac{8}{45}\right)$

21. Evaluate. Express each answer as a fraction in simplest (reduced) form.

a) $\left(\frac{1}{3}\right)^2$ b) $\left(\frac{3}{4}\right)^2$ c) $\left(\frac{2}{3}\right)^3$ d) $\left(-\frac{1}{5}\right)^2$ e) $\left(\frac{-4}{3}\right)^3$ f) $\left(4\frac{2}{3}\right)^2$ g) $\left(-3\frac{1}{2}\right)^4$

22. Evaluate. Express each answer as a mixed number, if applicable, and use technology to verify your answers.

a) $\frac{3}{4}\left(\frac{1}{2} - \frac{4}{5}\right)$ b) $\left(\frac{4}{3} - \frac{1}{5}\right)\left(\frac{3}{2} + \frac{5}{6}\right)$ c) $\left(-\frac{5}{6} + \frac{7}{12}\right)^2$ d) $\frac{5\frac{7}{8} - 2\frac{3}{4}}{2}$

e) $\frac{-\frac{5}{4} + 2}{\frac{3}{10}}$ f) $\left(-\frac{5}{12}\right)\left(-\frac{1}{4}\right) + \left(\frac{3}{2}\right)\left(\frac{8}{3}\right)$ g) $-\frac{1}{3}\left[1\frac{3}{4} - 6\frac{2}{3}\right]^2 + \left(\frac{5}{6}\right)^2$

23. Among other ingredients, a banana bread recipe that will serve 10 people calls for 3 bananas, $\frac{2}{3}$ cup of sugar and $1\frac{1}{2}$ cups of flour.

a) How much of each of these ingredients would be needed to serve twice as many people?

b) If a loaf that is $\frac{3}{2}$ the size of the given recipe is to be baked, how much of each of these ingredients is needed? How many people will it serve?

c) How much of each of these ingredients is needed to serve 24 people?

d) If $1\frac{1}{2}$ cups of sugar are used to make this banana bread, how many cups of flour should be used?

ANSWERS

CHECK YOUR UNDERSTANDING

1. $3\times\frac{1}{2}$ means three groups of $\frac{1}{2}$, which can be written as $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$ and is equal to $\frac{3}{2}$.

Alternatively, just as 3×2 means we have three twos, $3\times\frac{1}{2}$ means we literally have three halves, which is written as $\frac{3}{2}$.

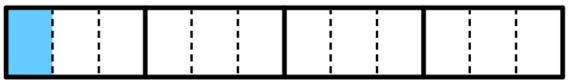
2. a) $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$ b) $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{3}{2}$ c) $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{10}{3}$ d) $\frac{7}{4}+\frac{7}{4}+\frac{7}{4}+\frac{7}{4}+\frac{7}{4}=\frac{35}{4}$

e) $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{6}{4}=\frac{3}{2}$ f) $\frac{5}{12}+\frac{5}{12}+\frac{5}{12}+\frac{5}{12}+\frac{5}{12}+\frac{5}{12}+\frac{5}{12}+\frac{5}{12}+\frac{5}{12}=\frac{45}{12}=\frac{15}{4}$

3. $\frac{3}{4}$ inch

4. a) $\frac{3}{5}$ b) $\frac{5}{8}$ c) $\frac{2}{9}$ d) $\frac{24}{11}$ e) $\frac{35}{16}$ f) $\frac{9}{4}$

5.



6. a) $\frac{1}{4}$ b) $\frac{1}{6}$ c) $\frac{3}{8}$ d) $\frac{1}{6}$ e) $-\frac{2}{15}$ f) $\frac{8}{15}$

7. a) $\frac{1}{8}$ b) $\frac{5}{18}$ c) $\frac{8}{9}$ d) $\frac{35}{27}$ e) $\frac{5}{12}$ f) $-\frac{3}{20}$

8. $3\div\frac{1}{4}$ is the number of quarters that go into 3. Since each whole (1) consists of 4 quarters, 3 wholes consists of 3×4 quarters. That is, there are 3×4 quarters in 3, and thus $3\div\frac{1}{4}=3\times4$.

9. a) 10 b) 12 c) $\frac{2}{3}$ d) $\frac{21}{8}$ e) $\frac{45}{8}$ f) $-\frac{5}{8}$

10. $\frac{3}{8}$

11. a) $\frac{35}{8}$ b) $\frac{20}{81}$ c) $\frac{7}{24}$ d) $-\frac{3}{2}$ e) $-\frac{15}{8}$ f) $\frac{7}{6}$

12. a) $\frac{4}{3}$ b) $\frac{12}{5}$ c) $\frac{15}{2}$ d) $\frac{3}{26}$ e) $-\frac{7}{24}$ f) $-\frac{2}{9}$

13. a) increase b) 2 times c) $\frac{1}{8}$ inch

14. a) $8\frac{4}{7}$ b) $10\frac{11}{15}$ c) $15\frac{1}{2}$ d) $-15\frac{47}{55}$ e) -23

15. Area $= 23\frac{4}{5}$ cm^2, Perimeter $= 19\frac{11}{15}$ cm

16. Area $= 6\frac{1}{64}$ inches^2

17. a) $2\frac{1}{5}$ b) $1\frac{5}{9}$ c) $-1\frac{13}{57}$ d) $1\frac{3}{13}$ e) $10\frac{1}{20}$

18. a) $3\frac{3}{5}$ hours b) $\frac{9}{10}$ hour c) $1\frac{2}{5}$ hours d) 1 hour and 48 minutes

19. $\frac{1257}{370}\times\frac{370}{2411}=\frac{1257\times\cancel{370}}{\cancel{370}\times 2411}=\frac{1257}{2411}$. The "cancellation" of the 370's here is actually the result of dividing both the numerator and denominator by 370, or dividing the 370 factor in the numerator by the 370 factor in the denominator.

20. a) $\frac{7}{20}$ b) $-\frac{15}{19}$ c) $\frac{10}{13}$ d) $\frac{57}{242}$ e) $\frac{6}{25}$

21. a) $\frac{1}{9}$ b) $\frac{9}{16}$ c) $\frac{8}{27}$ d) $\frac{1}{25}$ e) $-\frac{64}{27}$ f) $\frac{196}{9}$ g) $\frac{2401}{16}$

22. a) $-\frac{9}{40}$ b) $2\frac{29}{45}$ c) $\frac{1}{16}$ d) $1\frac{9}{16}$ e) $2\frac{1}{2}$ f) $4\frac{5}{48}$ g) $-7\frac{157}{432}$

23. a) 6 bananas, $1\frac{1}{3}$ cups of sugar and 3 cups of flour

b) $4\frac{1}{2}$ bananas, 1 cup of sugar and $2\frac{1}{4}$ cups of flour. Serves 15 people.

c) $7\frac{1}{5}$ bananas, $1\frac{3}{5}$ cups of sugar and $3\frac{3}{5}$ cups of flour

d) $3\frac{3}{8}$ cups of flour

ANSWERS