

MTH 1W: Multiplying and Dividing Fractions and Mixed Numbers

LESSON OVERVIEW:

Scope and Sequence – Main Lesson Topics	Prior Knowledge	Vocabulary
Multiplying and Dividing Fractions and Mixed Numbers <ul style="list-style-type: none"> • multiplying and dividing fractions and mixed numbers • powers of fractions and mixed numbers • order of operations involving fractions and mixed numbers 	<ul style="list-style-type: none"> • Integers • Order of operations • Powers 	<ul style="list-style-type: none"> • Unit Fractions • Benchmark Fractions • Common Fractions • Improper Fractions • Mixed Numbers

Learning Objectives	Curriculum Expectations
I can: <ul style="list-style-type: none"> • Multiply and divide positive and negative fractions • Use the order of operations with fractions and mixed numbers • I can evaluate powers of fractions and mixed numbers 	<ul style="list-style-type: none"> • B3.2 apply an understanding of unit fractions and their relationship to other fractional amounts, in various contexts, including the use of measuring tools • B3.3 apply an understanding of integers to explain the effects that positive and negative signs have on the values of ratios, rates, fractions, and decimals, in various contexts • B3.4 solve problems involving operations with positive and negative fractions and mixed numbers, including problems involving formulas, measurements, and linear relations, using technology when appropriate • E1.3 solve problems involving different units within a measurement system and between measurement systems, including those from various cultures or communities, using various representations and technology, when appropriate

1	Lesson Introduction & Problem String (<i>see below</i>)	40 minutes
Introduction: Mixed Numbers – a number that combines an integer and a fraction. E.g., $-8 \frac{1}{2}$ The integer in a mixed number can be re-written as an improper fraction with the same denominator as the mixed number. E.g., $-8 \frac{1}{2} = -16/2 + 1/2$ or $-17/2$ Review - Order of operations – BEDMAS. Same rules apply to working with integers		

2	Consolidation	10 minutes
<ul style="list-style-type: none"> • 		

3	Meaningful Notes	10 minutes
<ul style="list-style-type: none"> • Multiply fractions – multiply numerators and multiply denominators • Divide fractions – invert the second fraction and multiply • Integer Operations – multiply and divide integers = same signs answer is positive, opposite signs answer is negative 		

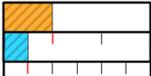
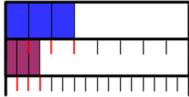
4	Check Your Understanding	15 minutes
<ul style="list-style-type: none"> • 		

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LESSON BACKGROUND:

- Operations with fractions have the same meaning as operations with whole numbers even though the algorithms are different. There are many models and/or procedures for computing with fractions just like there are with whole numbers.

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PROBLEM	HINTS	EXTENSIONS
$3 \times \frac{1}{4} =$	<p style="text-align: center;">$\frac{1}{4}$</p> <ul style="list-style-type: none"> Think of the question as: 3 sets of $\frac{1}{4}$ What does $\frac{1}{4}$ look like? How might you write this as repeated addition? Could you draw a picture? How might an area model help you visualize this? 	<ul style="list-style-type: none"> If it takes Jan $\frac{1}{4}$ hour to walk to music lessons. She has music once a week for three weeks. How long does Jan spend walking to her music lessons? Can you create a story problem similar to this one that uses different whole numbers and fractions?
$\frac{1}{3} \text{ of } 12$	<ul style="list-style-type: none"> Think of the question as ____ set of ____ What does $\frac{1}{3}$ of 1 look like? Can you draw or model it? What does $\frac{1}{3}$ of 2 look like? $\frac{1}{3}$ of 3? If I said 3 groups of 12, what would we do? This says "$\frac{1}{3}$ groups of 12" Could you draw a diagram to help you? 	<ul style="list-style-type: none"> How does the word "of" compare to multiplying? Does multiplication always make things bigger? Explain why or why not. Can you find different ways to model this problem?
$\frac{1}{2} \text{ of } \frac{1}{3}$	<ul style="list-style-type: none"> Think of the question as ____ set of ____ How many ____ in ____? Could you draw a picture to help you? Could you use tools (number line, fraction tower, relational rods) to help you? If 2×3 means 2 (sets) of 3, then what do you think $\frac{1}{2}$ of $\frac{1}{3}$ means? If the orange represents $\frac{1}{3}$ of the whole and the blue is $\frac{1}{2}$ of that $\frac{1}{3}$. What fraction of the whole does the blue represent? 	<ul style="list-style-type: none"> Draw this out. What does it look like? What fraction do we end up with? What is the relationship between the fraction you found (the product of that multiplication) and the denominators? What is the relationship between the fraction you found (the product of that multiplication) and the numerators? How might you solve a fraction problem if you were to use fractions that are not unit fractions?
<p>Evaluate. $\frac{3}{8} \times \frac{1}{2}$</p>	<ul style="list-style-type: none"> Think of the question as ____ set of ____ How many ____ in ____? What does multiplication mean? One of the things it means is "of". e.g., $3 \times 4 = 3$ sets of 4 How can you think of this problem as ____ sets of ____ Another way to think of fraction multiplication is to use fraction strips like these:  <ul style="list-style-type: none"> The purple bar is $\frac{1}{2}$ of the blue bar. If the blue bar is $\frac{3}{8}$ what is the value of the purple bar? 	<p style="text-align: center;">$\frac{3}{4} \times \frac{4}{3}$</p> <ul style="list-style-type: none"> This pair of fractions $\frac{3}{4} \times \frac{4}{3}$ has a product of 1. Change only one numerator or denominator each time to write a pair of fractions that has a product of: 2, of 3, of 4, and of 5. How could you write a pair of fractions that has a product of 10? Paula has $\frac{7}{8}$ of a tank of gas. She estimates that she will use $\frac{2}{3}$ of the gas to get home. What fraction of a tank of gas does she use?

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PROBLEM	HINTS	EXTENSIONS
	<ul style="list-style-type: none"> Another meaning of multiplication is that it can represent the area of a rectangle with given dimensions. Can you show how to multiply two fractions using an area model? 	
$2\frac{1}{2} \times 3\frac{1}{3} =$	<ul style="list-style-type: none"> Think of the question as ____ set of ____ How is this similar to the previous question? How is it different? Would equivalent, improper fractions help here? 	<ul style="list-style-type: none"> Why can't you simply multiply the two whole numbers and then multiply the fractions? Some products are in simplest form after you multiply. Some products are not in simplest form. How can you tell if a product of two fractions will be in simplest form after you multiply? Use examples to help you explain your thinking One way to solve multiplication of mixed numbers is to change them to equivalent improper fractions. Another way is to use an area model. Try to solve this problem using an area model. Which method do you prefer? Why? Do you think there are any times which it would make sense to use one method over the other?
$4 \div \frac{1}{2}$	<ul style="list-style-type: none"> Can you think of division as sharing? How many $\frac{1}{2}$ are in 4? What does that actually mean? What is that question actually asking you to do? 	<ul style="list-style-type: none"> Why did result increase from 4? When you divide a whole number by a proper fraction, is the quotient greater than or less than the whole number? Include examples in your explanation. $4 \div \frac{1}{2} \qquad \frac{1}{2} \div 4$ <ul style="list-style-type: none"> Why is $4 \div \frac{1}{2}$ not the same as $\frac{1}{2} \div 4$ (consider using number lines in your explanation) Inverting and multiplying is one algorithm that can be used when dividing two fractions. Another algorithm that can be used to divide two fractions is using common denominators. Try solving this problem using common denominators. Compare the two methods. When might you use one method over the other?
<p>How is $3 \div \frac{1}{4}$ the same as 3×4</p>	<ul style="list-style-type: none"> How many ____ are in ____? What is the same in these two problems? What is different? What do you notice about the number of pieces you have in total? 	<ul style="list-style-type: none"> When we multiply whole numbers $3 \times 4 = 12$ the answer increases. When we divide a whole number by a fraction, why did the result increase from 3? $\square \div \frac{\square}{\square}$

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	<ul style="list-style-type: none"> What is the relationship between the total number of pieces and the whole number and denominator? 	<p>a) Write the digits 2,4, and 6, in the boxes to find as many division statements as possible.</p> <p>b) Which statement in part a has the greatest quotient? The lease quotient? How do you know?</p>
$4 \div \frac{2}{3}$	<ul style="list-style-type: none"> How is the similar to the questions that we have done already? How is it different? What do you notice about the number of pieces you have in total? What is the relationship between the total number of pieces and the whole number and denominator? 	<p style="text-align: center;">$\frac{9}{2}$</p> <ul style="list-style-type: none"> The numbers $\frac{9}{2}$ and 3 share this property: Their difference is equal to their quotient. <p style="text-align: center;">$\frac{9}{2} - 3 = \frac{3}{2}$ $\frac{9}{2} \div 3 = \frac{3}{2}$</p> <ul style="list-style-type: none"> That is, $\frac{9}{2} - 3 = \frac{3}{2}$ and $\frac{9}{2} \div 3 = \frac{3}{2}$ Find other pairs of numbers with this property. Describe any patterns that you see.
$\frac{1}{2} \div \frac{1}{8}$	<p style="text-align: center;">$\frac{1}{8}$ $\frac{1}{2}$</p> <ul style="list-style-type: none"> What is this really asking? How many $\frac{1}{8}$ in $\frac{1}{2}$? How might knowing this, that, help you with an approach to solve? Do you think the answer will be greater than 1 or less than 1? Why do you think that? Could you draw a picture or use a model to help you represent this problem? 	<ul style="list-style-type: none"> How does a visual representation of this problem compare to the algorithm? Write as many division statements as you can that have $\frac{5}{6}$ as their quotient When you divide two fractions, how can you tell, before you divide, if the quotient will be: <ul style="list-style-type: none"> Greater than 1? Less than 1? Equal to 1? Use examples in your explanations
$4\frac{2}{3} \div 1\frac{1}{2}$	<p style="text-align: center;">$\frac{1}{2} \div \frac{1}{8}$</p> <ul style="list-style-type: none"> How is this similar to the previous question? How is it different? How might you make these mixed numbers easier to work with? What do you think this question is asking? (How many $1\frac{1}{2}$ are in $4\frac{2}{3}$) 	<ul style="list-style-type: none"> Explain how you know that your answer makes sense. What are some areas that other students might find tricky or challenging? What hints might you give them to help them solve the problem without telling them what to do? Inverting and multiplying is one algorithm that can be used when dividing two fractions. Another algorithm that can be used to divide two fractions is using common denominators. Try solving this problem using common denominators. Compare the two methods. When might you use one method over the other?

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Compare $\left(\frac{2}{3}\right)^2$ and $\left(\frac{-2}{3}\right)^2$	<ul style="list-style-type: none"> How might we represent $(2/3)^2$ as repeated multiplication? What do you recall about working with integers from last year? What happens if you multiply 2 positive integers? 2 negative integers? A negative and a positive integer? 	<ul style="list-style-type: none"> How would these fractions compare if they were written to the power 3? What are some areas that you think other students might find tricky or challenging? What hints would you give them to help them solve the problem without telling them what to do? If you were to write a note to your "future forgetful self" about solving this type of problem, what information would you include? Discuss in your group what you think the most important ideas are
Evaluate. $\left(2\frac{1}{3}\right)^2$	<ul style="list-style-type: none"> How might we represent this problem as repeated multiplication What do you think you might need to do with the 	<ul style="list-style-type: none"> Explain why your answer makes sense. What are some areas that other students might find tricky or challenging? What hints might you give them to help them solve the problem without telling them what to do? If you were to write a note to your "future forgetful self" about solving this type of problem, what information would you include? Discuss in your group what you think the most important ideas are
$\left(\frac{3}{5} + \frac{1}{15}\right) \times \frac{2}{3}$	<ul style="list-style-type: none"> How might BEDMAS help you? What operation would you do first? Why that one? What are you going to do with the fractions that need to be added? 	<ul style="list-style-type: none"> Create some of your own multi-step fraction problems. What are some areas that you think other students might find tricky or challenging? What hints would you give them to help them solve the problem without telling them what to do? If you were to write a note to your "future forgetful self" about solving this type of problem, what information would you include? Discuss in your group what you think the most important ideas are

