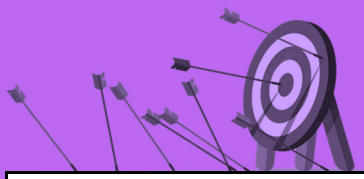
ADDING AND SUBTRACTING POLYNOMIALS

BIG IDEAS:

- Sums and differences of monomials are called **polynomials**
- **Monomials** themselves are also **polynomials**
- Polynomials can be added and subtracted by **collecting like terms**
- A polynomial can be **subtracted** by **adding the opposites** of its terms

LEARNING GOALS AND SKILL DEVELOPMENT:
You know you have met the goals for this lesson when you can:



EMERGING

LEARNING GOALS	ANCHOR QUESTIONS
Add a binomial to a monomial	3
Subtract a binomial from a binomial	4

SKILL BUILDING QUESTIONS			
1	2	3	4

EVOLVING

LEARNING GOALS	ANCHOR QUESTIONS
Simplify expressions by adding and subtracting polynomials	5, 9
Write simplified expressions to represent the perimeter of a rectangle or triangle	12
Solve real-world problems by writing and simplifying polynomial expressions	13

SKILL BUILDING QUESTIONS			
5	6	7	8
9	10	11abc	12
13	17		

EXTENDING

LEARNING GOALS	ANCHOR QUESTIONS
Simplify expressions by adding and subtracting polynomials, including terms with fractional coefficients	11d
Solve real-world problems by writing and simplifying polynomial expressions including terms where the variable has an exponent of 2	15
Write simplified expressions to represent the unknown length of a side in a rectangle or triangle when given the perimeter	18

SKILL BUILDING QUESTIONS			
11d	14	15	16
18	19	20	

BUILD YOUR SKILLS

1. Explain why $3+(4+5)$ is equal to $3+4+5$.

2. Explain why $10-(4+3)$ is equal to $10-4-3$.

3. Add.

a) $3x+(4x+2)$ b) $5+(6x-3)$ c) $(5x+2)+(3x+8)$ d) $(2x-9)+(-5x+1)$

4. Subtract.

a) $7x-(2x+3)$ b) $5-(2x-3)$ c) $(6x+8)-(2x+5)$ d) $(6x-5)-(-8x+10)$

5. Simplify.

a) $(-4y+6)+(-12y-8)$ b) $\left(x^2+2x+5\right)+\left(x^2+6x+8\right)$ c) $\left(4a^2-3a\right)-\left(6a^2+4a\right)$

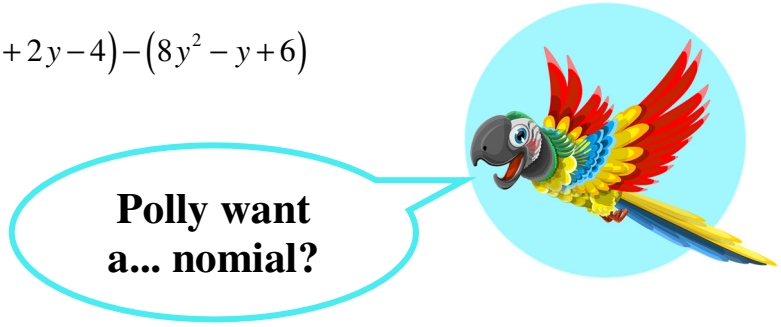
d) $\left(3m^2+7m-6\right)-\left(-m^2-4m+5\right)$ e) $\left(-9y^2-11y+2\right)+\left(3+9y^2\right)$

6. Explain why $2+(7-3)$ is equal to $2+7-3$.

7. We can think of subtracting as "adding the opposite." For example, $(2x+3)-(4x-5)$ can be expressed as $(2x+3)+(-4x+5)$. For each of the following, express the subtraction of polynomials as an addition and then evaluate the sum.

a) $(6x+2)-(-5x+8)$ b) $\left(3y^2+2y-4\right)-\left(8y^2-y+6\right)$

c) $\left(-a-3\right)-\left(-2a^2+3a-4\right)$



8. While simplifying the expression $10x^2-\left(3x^2-4x+5\right)$, Margot believed she had found a shortcut for subtracting polynomials. She said,

"I can just remove the brackets and change the sign of each term of the polynomial I'm subtracting!
So, this expression is the same as $10x^2-3x^2+4x-5$*."*

Is Margot's claim correct? Explain.

9. Simplify.

a) $(5x+4)-7x$

b) $(-4n+2)-(9n-7)$

c) $\left(4x^2-7x+5\right)+\left(6x^2+9x-8\right)$

d) $\left(7p^2-10p+19\right)-\left(-6p^2-4p+12\right)$

e) $\left(y^3+4y^2+2\right)+\left(6y^2-3y+11\right)$

f) $\left(7-3x-4x^2\right)+\left(9x^2+x\right)$

g) $\left(5z-9\right)-\left(9z-8+z^2\right)$

h) $(a+b-c)+(4b-c+3d)$

10. Simplify and evaluate for $x=2$ and $y=-3$.

a) $(7x+3)-(-2x-4)$

b) $17y^2-\left(15y^2+3y-8\right)$

c) $\left(8x-4y\right)-\left(3y+6x\right)$

11. Simplify.

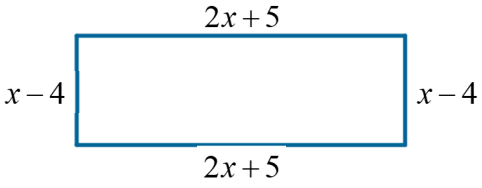
a) $(5x+3)+(2x-1)+(-3x+8)$

b) $(3a+4)-(a-6)+(-4a+7)$

c) $\left(3xy^2+6x-8y\right)+\left(9xy^2-2x+4y\right)$

d) $\left(\frac{1}{2}n^2+\frac{2}{3}n-\frac{5}{3}\right)-\left(\frac{3}{8}n^2-\frac{3}{4}n+\frac{7}{3}\right)$

12. A rectangle is shown on the right with algebraic expressions that represent the lengths of its sides. Determine a simplified expression that represents the perimeter of the rectangle.



13. To perform music at an event, Elise charges an initial fee of \$100 and a rate of \$85 per hour of work on the event day. Therefore, her earnings for a booking can be modelled by the expression $100+85t$, where t represents the number of hours spent working on the day of the event. To photograph an event, Deion charges \$70 per hour worked at the event and a one-time photo processing fee of \$310.

a) State an expression to represent Deion's total earnings for a booking that is t hours long.

b) Assuming the Elise and Deion both worked for t hours at an event, express their combined earnings from the booking as a sum of polynomials.

c) Simplify your expression from part (b).

d) Using your expression from part (c), determine Elise's and Deion's combined earnings for a 5-hour booking.

e) Assuming that no booking is longer than 8 hours, who earns more money for an event that is t hours long?

f) Determine a simplified expression to represent the difference between Elise's and Deion's earnings for an event at which they both worked t hours.

g) In order for Elise and Deion to make the exact same total earnings for working the same number of hours at an event, for how long would they theoretically need to work at the event?

14. When using *column form* to add or subtract polynomials, like terms are aligned vertically. The following example illustrates that $(3x^2+5x-4)+(x^2-9x+7)$ equals $4x^2-4x+3$.

$$\begin{array}{r} 3x^2+5x-4 \\ +\ x^2-9x+7 \\ \hline 4x^2-4x+3 \end{array}$$

Express each of the following in column form and then add or subtract, as indicated.

a) $(-4x+6)+(6x-7)$

b) $(y^2+5y-8)+(2y^2-7y+4)$

c) $(8x-4)-(3x+2)$

d) $(6a-3)+(4-7a)$

e) $(-x^2+5x-7)-(-2x^2+4x-3)$

f) $(-3y^2+4y+2)+(6y^2-8)$

g) $(8x^2+3x)-(2x^2-4x+5)$

h) $(8a+5-6a^2)+(2-4a^2)$

15. The height, in metres, of a golf ball above the ground t seconds after it is hit is given by the expression $-4.9t^2 + 21.8t$. Similarly, the height of a baseball that is hit at the exact same time as the golf ball is represented by the expression $-4.9t^2 + 20.4t + 2.4$.

a) Interpret the expression $\left(-4.9t^2 + 20.4t + 2.4\right) - \left(-4.9t^2 + 21.8t\right)$ in the context of this situation.

b) Simplify the expression from part (a).

c) Evaluate your result from part (b) for $t = 1$ and $t = 3$.

d) The two values calculated in part (c) have different signs. Interpret these opposite signs in the context of this situation.

16. Simplify

a) $\left(7a^2b - 6ab + 5ab^2\right) + \left(6a^2b + 2ab - 4ab^2\right)$

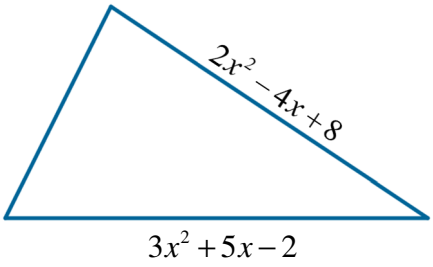
b) $\left(8x^2y^2 - 4x^2y + 6xy^2\right) - \left(-3x^2y^2 + 4xy^2\right)$

c) $\left(14x^3y^2 - 8x^3 + 9y^2\right) - \left(5x^3 + 4y^3 + 6x^3y^2\right)$

d) $\left(\frac{5}{4}g^2 + 2g - \frac{2}{3}\right) + \left(2 - \frac{3}{7}g + \frac{g^2}{3}\right)$

17. A rectangle's length is represented by the expression $6x + 5y$ and its width can be expressed as $4x - 3y$. Determine a simplified expression to represent the perimeter of the rectangle.

18. For the triangle shown below, algebraic representations are given for the lengths of two sides. If the perimeter of the triangle is $2x^2 + x + 13$, determine a simplified expression for the length of the third side.



19. Javier claims that it is possible for the sum of two binomials to result in a monomial. Is Javier's claim correct? Explain.

20. Alia claims that it is possible for the difference of two binomials to result in a trinomial. Is Alia's claim correct? Explain.

ANSWERS

CHECK YOUR UNDERSTANDING

1. Adding the sum of 4 and 5 to 3 gives the same result as adding 4 and 5 to 3 successively. In both cases, 9 is ultimately added to 3.

2. Subtracting the sum of 4 and 3 from 10 gives the same result as subtracting 4 and 3 from 10 successively. In both cases, 7 is ultimately subtracted from 10.

3. a) $7x+2$ b) $6x+2$ c) $8x+10$ d) $-3x-8$

4. a) $5x-3$ b) $-2x+8$ c) $4x+3$ d) $14x-15$

5. a) $-16y-2$ b) $2x^2+8x+13$ c) $-2a^2-7a$ d) $4m^2+11m-11$ e) $-11y+5$

6. Adding the difference of 7 and 3 to 2 gives the same result as first adding 7 to 2 and then subtracting 3 from the result. In both cases, 4 is ultimately added to 2.

7. a) $(6x+2)+(5x-8)\,;\,11x-6$ b) $\left(3y^2+2y-4\right)+\left(-8y^2+y-6\right)\,;\,-5y^2+3y-10$

c) $\left(-a-3\right)+\left(2a^2-3a+4\right)\,;\,2a^2-4a+1$

8. Yes. This shortcut is essentially the same as adding the opposite. That is, $10x^2-\left(3x^2-4x+5\right)$ is equivalent to $10x^2+\left(-3x^2+4x-5\right)$, which can be written as $10x^2-3x^2+4x-5$ and then simplified.

9. a) $-2x+4$ b) $-13n+9$ c) $10x^2+2x-3$ d) $13p^2-6p+7$ e) $y^3+10y^2-3y+13$

f) $5x^2-2x+7$ g) $-z^2-4z-1$ h) $a+5b-2c+3d$

10. a) $9x+7\,;\,25$ b) $2y^2-3y+8\,;\,35$ c) $2x-7y$

11. a) $4x+10$ b) $-2a+17$ c) $12xy^2+4x-4y$ d) $\frac{1}{8}n^2+\frac{17}{12}n-4$

12. $6x+2$

13. a) $310+70t$ b) $(100+85t)+(310+70t)$ c) $155t+410$ d) \$1185 e) Deion

f) $-15t+210$ g) 14 hours

14. a) $\begin{array}{r} -4x+6 \\ +\ 6x-7 \\ \hline 2x-1 \end{array}$ b) $\begin{array}{r} y^2+5y-8 \\ +\ 2y^2-7y+4 \\ \hline 3y^2-2y-4 \end{array}$ c) $\begin{array}{r} 8x-4 \\ -\ 3x+2 \\ \hline 5x-6 \end{array}$ d) $\begin{array}{r} 6a-3 \\ +\ -7a+4 \\ \hline -a+1 \end{array}$

e) $\begin{array}{r} -x^2+5x-7 \\ -\ -2x^2+4x-3 \\ \hline x^2+x-4 \end{array}$ f) $\begin{array}{r} -3y^2+4y+2 \\ +\ 6y^2+0y-8 \\ \hline 3y^2+4y-6 \end{array}$ g) $\begin{array}{r} 8x^2+3x+0 \\ -\ 2x^2-4x+5 \\ \hline 6x^2+7x-5 \end{array}$ h) $\begin{array}{r} -6a^2+8a+5 \\ +\ -4a^2+0a+2 \\ \hline -10a^2+8a+7 \end{array}$

15. a) The expression represents the height of the baseball relative to the golf ball.

b) $-1.4t+2.4$

c) 1 for $t=1$ and -1.8 for $t=3$

d) The positive value for $t=1$ indicates that the baseball is higher than the golf ball at a time of 1 second. The negative value for $t=3$ indicates that the baseball is lower than the golf ball at a time of 3 seconds.

16. a) $13a^2b-4ab+ab^2$ b) $11x^2y^2-4x^2y+2xy^2$ c) $8x^3y^2-13x^3-4y^3+9y^2$

d) $\frac{19}{12}g^2+\frac{11}{7}g+\frac{4}{3}$

17. $20x+4y$

18. $-3x^2+7$

19. Yes. For example, $(2x+7)+(3x-7)$, which gives a result of $5x$.

20. Yes. For example, $(4x^2+5)-(7x+3)$, which gives a result of $4x^2-7x+2$.

ANSWERS