ADDING AND SUBTRACTING FRACTIONS AND MIXED NUMBERS

BIG IDEAS:

- Positive fractions can be written as a **sum of unit fractions**
- A **common denominator** is used for **adding** and **subtracting** fractions
- **Mixed numbers** can be added and subtracted by **converting to improper fractions** or by looking at the **whole part and fraction part** separately

LEARNING GOALS AND SKILL DEVELOPMENT:
You know you have met the goals for this lesson when you can:

EMERGING

LEARNING GOALS	ANCHOR QUESTIONS
Add unit fractions	2
Write a fraction as a sum of unit fractions	3
Add fractions using a visual representation with common denominators or different denominators where one denominator is a factor of the other	4

SKILL BUILDING QUESTIONS			
1	2	3	4
6			

EVOLVING

LEARNING GOALS	ANCHOR QUESTIONS
Add or subtract positive and negative fractions with common or different denominators	7, 8
Solve real-world problems involving addition and subtraction of fractions with common and different denominators	9, 14

SKILL BUILDING QUESTIONS			
5	7	8	9
10	11	14	15
17	18		

EXTENDING

LEARNING GOALS	ANCHOR QUESTIONS
Add or subtract positive and negative fractions and mixed numbers with common or different denominators	12, 17

SKILL BUILDING QUESTIONS			
12	13	16	19
20	21	22	

BUILD YOUR SKILLS

1. Write an addition or subtraction statement that could be represented by each of the following.

a)

b)

2. Express each sum of unit fractions as a single fraction.

a) $\frac{1}{3}+\frac{1}{3}$ b) $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$ c) $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ d) $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$

3. Express each of the following as a sum of unit fractions.

a) $\frac{3}{5}$ b) $\frac{4}{7}$ c) $\frac{3}{2}$ d) $2\frac{1}{4}$

4. Use a visual representation to illustrate each of the following.

a) $\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$ b) $\frac{1}{3}+\frac{2}{9}=\frac{5}{9}$ c) $\frac{5}{8}-\frac{3}{8}=\frac{1}{4}$ d) $\frac{5}{6}-\frac{2}{3}=\frac{1}{6}$

5. Explain the purpose of a common denominator when adding or subtracting fractions.

6. Evaluate.

a) $\frac{3}{17}+\frac{9}{17}$ b) $\frac{6}{7}-\frac{4}{7}$ c) $\frac{2}{3}+\frac{5}{3}$ d) $\frac{1}{5}-\frac{4}{5}$ e) $-\frac{2}{9}+\frac{7}{9}$ f) $\frac{4}{3}+\left(-\frac{14}{3}\right)$

7. Add.

a) $\frac{1}{2}+\frac{1}{4}$ b) $\frac{2}{3}+\frac{2}{9}$ c) $\frac{1}{2}+\frac{1}{3}$ d) $\frac{3}{10}+\frac{2}{7}$ e) $-\frac{2}{5}+\frac{5}{6}$ f) $\frac{7}{8}+\left(-\frac{4}{3}\right)$

8. Subtract.

a) $\frac{1}{3}-\frac{1}{6}$ b) $\frac{3}{5}-\frac{7}{15}$ c) $\frac{4}{5}-\frac{5}{8}$ d) $\frac{1}{4}-\frac{1}{3}$ e) $\frac{3}{8}-\frac{5}{9}$ f) $\frac{2}{7}-\left(-\frac{1}{3}\right)$

9. Aiguo, Destiny and Claudio shared a 12-slice pizza. Aiguo ate $\frac{1}{3}$ of the pizza and Destiny ate $\frac{1}{4}$ of the pizza. Claudio ate the remaining slices.

a) What fraction of the pizza did Aiguo and Destiny eat together?

b) What fraction of the pizza did Claudio eat?

c) How many slices did each person eat?

10. Evaluate. Express each answer as a fraction in lowest terms.

a) $\frac{2}{9}+\frac{1}{3}$ b) $\frac{3}{4}-\frac{5}{12}$ c) $\frac{4}{11}+\frac{1}{3}$ d) $\frac{5}{6}-\frac{3}{8}$ e) $\frac{11}{4}-\frac{3}{2}$ f) $-\frac{4}{5}+\frac{1}{3}$

g) $\frac{5}{3}-\frac{9}{2}$ h) $\frac{11}{12}+\left(-\frac{9}{8}\right)$ i) $5-\frac{17}{6}$ j) $\frac{11}{18}-\left(-\frac{7}{12}\right)$ k) $\frac{-7}{4}+\left(-\frac{15}{6}\right)$

11. Of the students in a class, $\frac{3}{4}$ take the bus to school. $\frac{3}{16}$ of the students in the class walk to school. The remaining students are driven to school in a car.

a) What fraction of the class is driven to school in a car?

b) What fraction of the class does not walk to school?

c) Is it possible that there is a total of 25 students in the class? Explain.

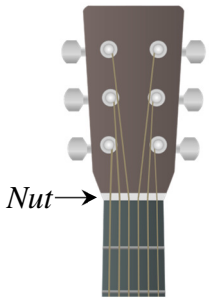
12. Between which two consecutive whole numbers will the result of each of the following lie?

a) $12+10\frac{4}{9}$ b) $8\frac{1}{5}+3\frac{2}{5}$ c) $14\frac{1}{2}+25\frac{3}{4}$ d) $9\frac{3}{4}-7\frac{2}{3}$ e) $19\frac{3}{8}-14\frac{5}{7}$

13. Evaluate the sums and differences in question #12. Express each answer as a mixed number.

14. A gingerbread cookie recipe calls for $\frac{3}{4}$ cup of sugar, 2 cups of flour, $\frac{1}{3}$ cup of dark molasses, $\frac{1}{2}$ cup of water and $\frac{2}{3}$ cup of shortening. Determine the combined amount, in cups, of these ingredients. Express your answer as a mixed number.

15. On a steel-string acoustic guitar, the material that supports the strings at the end closest to the headstock is called the *nut*. The width of the nut typically ranges between $1\frac{11}{16}$ inches and $1\frac{3}{4}$ inches. Determine the difference between these two nut widths.



16. Samir plans to attach a shelf to his bedroom wall. In order to support the heavy load that will be placed on the shelf, he'll be fastening it to the wooden studs behind the wall using $2\frac{1}{2}$" screws. Each screw will pass through a section of the shelf that is $\frac{7}{8}$" thick and a $\frac{1}{2}$" sheet of drywall before reaching the stud. Determine, as a mixed number, how far each screw will rest in the stud?

17. Evaluate. Express each answer as a mixed number.

a) $3\frac{4}{5}+\left(-1\frac{7}{10}\right)$ b) $2\frac{2}{7}-4\frac{3}{5}$ c) $-5\frac{1}{6}+3\frac{2}{3}$ d) $10\frac{8}{15}+2$ e) $-4\frac{5}{6}-10\frac{6}{7}$

f) $\frac{-5}{3}+5\frac{6}{11}$ g) $7-12\frac{3}{4}$ h) $-8+\left(\frac{11}{4}\right)$ i) $\frac{15}{-6}-2\frac{7}{9}$ j) $-3\frac{5}{6}-\left(-4\frac{3}{8}\right)$

18. On a road trip, four friends shared the driving. Kiley did $\frac{1}{3}$ of the driving and Rami did $\frac{3}{8}$ of the driving. Since Ryan wasn't feeling well, he only did $\frac{1}{12}$ of the driving. The remainder of the driving was done by Melanie.

a) Determine the total fraction of the trip that was driven by Kiley, Rami and Ryan.

b) What fraction of the trip was driven by Melanie?

c) If the total time driving was 11 hours and 12 minutes, determine how long each person spent driving. Express each answer as a combination of hours and minutes, where applicable.

19. Evaluate.

a) $2\frac{7}{9}+1\frac{2}{3}-\frac{4}{5}$ b) $\frac{7}{16}+\left(\frac{5}{8}-\frac{3}{4}\right)$ c) $10\frac{3}{5}-\left(5\frac{1}{4}+2\frac{2}{3}\right)$ d) $-9\frac{7}{10}-\left(2\frac{5}{6}-4\frac{2}{9}\right)$

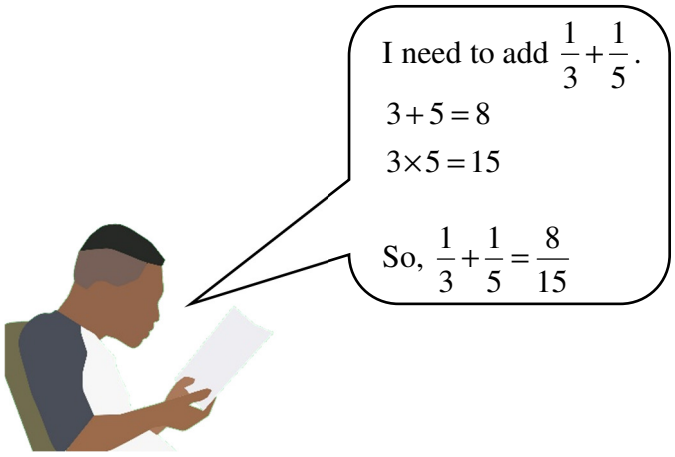
20. When adding mixed numbers, can the whole parts and the fraction parts be added separately and then recombined? Explain.

21. When subtracting mixed numbers, can the whole parts and the fraction parts be handled separately? Explain.

22. While adding unit fractions, Jamal believed he had discovered a shortcut. He hypothesized that when adding two unit fractions, he could determine the resulting fraction's numerator by adding the original two denominators, and he could find the resulting fraction's denominator by multiplying the original two denominators.

a) Prove that Jamal's hypothesis is correct.

b) What are some drawbacks of Jamal's shortcut?



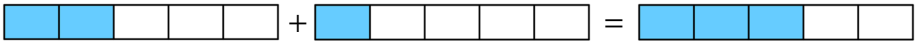
ANSWERS

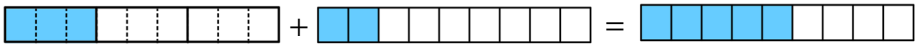
CHECK YOUR UNDERSTANDING

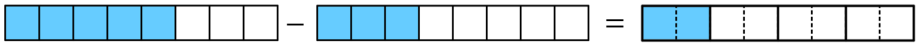
1. a) $\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$ b) $\frac{3}{4}-\frac{2}{3}=\frac{1}{12}$

2. a) $\frac{2}{3}$ b) $\frac{3}{5}$ c) $\frac{5}{8}$ d) $\frac{5}{4}$

3. a) $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$ b) $\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}$ c) $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$ d) $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$

4. a) 

b) 

c) 

d)

5. When adding and subtracting fractions, a common denominator allows us to operate using parts that have the same size. This idea is similar to the way we would prefer a common unit when working with measurements (cm, mm, inches, etc.).

6. a) $\frac{12}{17}$ b) $\frac{2}{7}$ c) $\frac{7}{3}$ d) $-\frac{3}{5}$ e) $\frac{5}{9}$ f) $-\frac{10}{3}$

7. a) $\frac{3}{4}$ b) $\frac{8}{9}$ c) $\frac{5}{6}$ d) $\frac{41}{70}$ e) $\frac{13}{30}$ f) $-\frac{11}{24}$

8. a) $\frac{1}{6}$ b) $\frac{2}{15}$ c) $\frac{7}{40}$ d) $-\frac{1}{12}$ e) $-\frac{13}{72}$ f) $\frac{13}{21}$

9. a) $\frac{7}{12}$ b) $\frac{5}{12}$ c) Aiguo ate 4 slices, Destiny ate 3 slices and Claudio ate 5 slices.

10. a) $\frac{5}{9}$ b) $\frac{1}{3}$ c) $\frac{23}{33}$ d) $\frac{11}{24}$ e) $\frac{5}{4}$ f) $-\frac{7}{15}$ g) $-\frac{17}{6}$ h) $-\frac{5}{24}$ i) $\frac{13}{6}$ j) $\frac{43}{36}$ k) $-\frac{17}{4}$

11. a) $\frac{1}{16}$ b) $\frac{13}{16}$

c) No, since the number of students should be a whole number and, for example, $\frac{3}{4}$ of 25 is not a whole number.

12. a) 22 and 23 b) 11 and 12 c) 40 and 41 d) 2 and 3 e) 4 and 5

13. a) $22\frac{4}{9}$ b) $11\frac{3}{5}$ c) $40\frac{1}{4}$ d) $2\frac{1}{12}$ e) $4\frac{37}{56}$

14. $4\frac{1}{4}$ cups

15. $\frac{1}{16}$ inch

16. $1\frac{1}{8}$ inches

17. a) $2\frac{1}{10}$ b) $-2\frac{11}{35}$ c) $-1\frac{1}{2}$ d) $12\frac{8}{15}$ e) $-15\frac{29}{42}$ f) $3\frac{29}{33}$ g) $-5\frac{3}{4}$

h) $-5\frac{1}{4}$ i) $-5\frac{5}{18}$ j) $\frac{13}{24}$

18. a) $\frac{19}{24}$ b) $\frac{5}{24}$

c) Kiley drove for 3 hours and 44 minutes. Rami drove for 4 hours and 12 minutes. Ryan drove for 56 minutes. Melanie drove for 2 hours and 20 minutes.

19. a) $3\frac{29}{45}$ b) $\frac{5}{16}$ c) $2\frac{41}{60}$ d) $-8\frac{14}{45}$

20. Yes. For example, $2\frac{1}{5}+4\frac{3}{5}$ is equivalent to $2+\frac{1}{5}+4+\frac{3}{5}$, which can be rewritten as $2+4+\frac{1}{5}+\frac{3}{5}$. Adding in pairs gives $6+\frac{4}{5}$, which is equivalent to $6\frac{4}{5}$.

21. Yes. For example, $6\frac{4}{5}-2\frac{1}{5}$ is equivalent to $6+\frac{4}{5}-2-\frac{1}{5}$, which can be rewritten as $6-2+\frac{4}{5}-\frac{1}{5}$. Subtracting in pairs gives $4+\frac{3}{5}$, which is equivalent to $4\frac{3}{5}$.

ANSWERS

ANSWERS

22. a) $\frac{1}{a}+\frac{1}{b}$

$=\frac{1\times b}{a\times b}+\frac{1\times a}{b\times a}$

$=\frac{b}{ab}+\frac{a}{ba}$

$=\frac{b}{ab}+\frac{a}{ab}$

$=\frac{b+a}{ab}$

$=\frac{a+b}{ab}$

b) Jamal's shortcut will not always give the resulting fraction in lowest terms. Furthermore, his shortcut does not apply to addition in which one or both of the fractions are non-unit fractions.