RATES AND PERCENTAGES

BIG IDEAS:

- A **unit rate** is a comparison of quantities in which the second quantity is **1**
- A **percentage** is a ratio in which the second term is **100**
- **Proportions** can be used to solve problems involving rates and percentages
- Percentages are often converted to **decimal values** for calculations

LEARNING GOALS AND SKILL DEVELOPMENT:
You know you have met the goals for this lesson when you can:

EMERGING

LEARNING GOALS	ANCHOR QUESTIONS
Determine a unit rate	1, 2
Explain the meaning of percent	3
Write a fraction as a percent	4
Determine a percentage of a whole number	5

SKILL BUILDING QUESTIONS			
1	2	3	4
5	6		

EVOLVING

LEARNING GOALS	ANCHOR QUESTIONS
Solve real-world problems involving rates	7, 9, 13
Solve money problems involving rates and percentages (tax and discount)	11, 13

SKILL BUILDING QUESTIONS			
7	8	9	10
11	12	13	14

EXTENDING

LEARNING GOALS	ANCHOR QUESTIONS
Determine the percentage increase in a given situation	15
Solve problems involving percentages that make connections to other curriculum topics, such as area and volume	17, 18

SKILL BUILDING QUESTIONS			
15	16	17	18

BUILD YOUR SKILLS

1. Fill in the blanks to complete each unit rate. Do not round any values.

    a) $65 for 5 boxes = $____/box

    b) 24 cans for $10.80 = $____/can

    c) 9 boxes for $38.25 = $____/box

    d) 175 km in 2.5 hours = ____ km/h

    e) $25.50 for 204 boxes = ____boxes/dollar

    f) $6.75 for 750 mL = $____/mL

2. A box of Huggers diapers contains 198 diapers and costs $43.99. A box of Snuggies diapers contains 136 diapers and costs $24.89. Determine which brand offers a lower cost per diaper and by how much.

3. Explain the meaning of the word *percent*.

4. Express each of the following as a percentage.

    a) $\dfrac{73}{100}$ b) $\dfrac{42}{50}$ c) $\dfrac{7}{10}$ d) $\dfrac{18}{23}$ e) $\dfrac{90}{200}$ f) $\dfrac{38}{60}$ g) $\dfrac{67}{50}$ h) $\dfrac{48}{40}$

5. Determine the value of each of the following.

    a) 50% of 40   b) 25% of 40   c) 7% of 200   d) 35.4% of 430   e) 6.5% of 80

6. A fruit bowl contains only apples and oranges. The ratio of apples to oranges is $2:3$. What percentage of the fruits in the bowl are oranges?

7. A truck travels at a constant speed of 95 km/h. How long will it take for the truck to travel 304 km? Express your answer in minutes.

8. Basic Foods sells red bell peppers for $2.99/lb, whereas Foodworld sells them for $3.49/lb. If Jagoda needs to purchase 2.5 pounds of red bell peppers, how much will she save by shopping at Basic Foods?
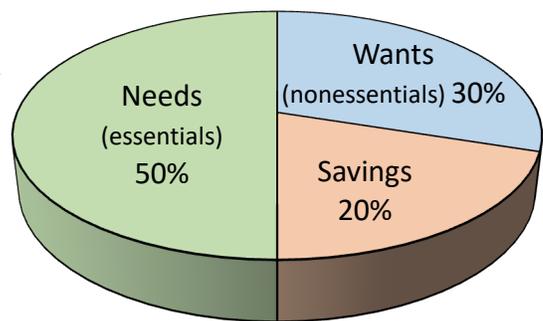*Note: Sales tax does not apply to vegetables.*

9. Ayobami can type 360 words in 8 minutes and Chayann can type 516 words in 12 minutes. Determine who types faster and by how many words per minute.
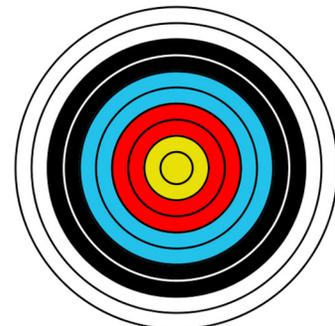
10. Thalia is considering purchasing a new pair of hockey skates, priced at $159.99. Determine the cost of the skates after a 13% sales tax is applied.

11. A computer that has a regular price of $895.99 is on sale for 20% off.

    a) Determine the price of the computer after the discount is applied (excluding tax).

    b) Determine the cost of the discounted computer after tax (13% HST).

    c) Does applying the tax before the discount affect the final cost for the customer? Explain.

12. Nanette's gross income is $59 384/year.

    a) Nanette's net income is $41 569/year. What percentage of her gross income is her take-home pay?

    b) The *50/30/20 rule* is a simple starting point for personal budgeting and suggests dividing net income as shown on the right.
    Using this guideline, determine how much Nanette should allocate to each category.

Wants (nonessentials) 30%
Needs (essentials) 50%
Savings 20%

13. A package of Powermate AA batteries costs $8.99 and contains 4 batteries. Electrosource AA batteries are sold in packages of 12 for $20.39.

    a) Which brand is a better buy?

    b) If Powermate batteries last twice as long as Electrosource batteries, which option is the better buy?

    c) What other factors should be considered before choosing one of the two options?

14. A circular archery target that is used in Olympic competition is shown below. The target has a diameter of 122 cm.

    a) The diameter of the innermost circle is 10% of the entire target's diameter. Determine the diameter of the innermost circle.

    b) The nine rings surrounding the innermost circle have equal width. Determine the width of each ring.

    c) What percentage of the total area are the two outer (white) rings?

15. *Percentage increase* is the difference between a final value and an initial value, expressed as a percentage of the initial value. For example, from 10 to 15 is a 50% increase, since the value increased by 5, which is 50% of 10. Find the percentage increase for the following.

    a) The price of a sweater is increased from $34.99 to $42.99.

    b) The cost of a photocopying service is increased from 2¢ per page to 4.5¢ per page.

    c) The area of a parking lot is tripled.

16. When Canadian guitarist Jeffrey McFadden plays the first movement of Enrique Granados' *Valses Poéticos* at a tempo of 120 beats per minute (bpm), its duration is approximately 1 minute and 10 seconds. Determine the movement's duration when practised by a student at a slower tempo of 88 bpm.

17. A square has a side length of 5 cm.

    a) If the side length is doubled, what percentage of the original area is the new area?

    b) Prove that the result from part (a) is true for all squares.

18. A cube has an edge length of 4 inches.

    a) If the edge length is tripled, what percentage of the original volume is the new volume?

    b) Prove that the result from part (a) is true for all cubes.

CHECK YOUR UNDERSTANDING

1. a) $13/box   b) $0.45/can   c) $4.25/box   d) 70 km/h   e) 8 boxes/dollar

f) $0.009/mL

2. Snuggies, by approximately $0.04/diaper.

3. Percent means per 100, or in other words, out of 100.

4. a) 73%   b) 84%   c) 70%   d) 78.3%   e) 45%   f) 63.3%   g) 134%   h) 120%

5. a) 20   b) 10   c) 14   d) 152.22   e) 5.2

6. 60%

7. 192 minutes

8. $1.25

9. Ayobami, by 2 words/minute.

10. $180.79

11. a) $716.79   b) $809.97

c) No. The discounted price can be found by multiplying the regular price by 0.8, since the costumer pays 80% of the regular price. Multiplying the result by 1.13 gives the total cost after tax. So,

$$\text{Final cost} = 1.13 \times (0.8 \times 895.99) \leftarrow \text{discount applied first}$$
$$= 1.13 \times 0.8 \times 895.99$$
$$= 0.8 \times 1.13 \times 895.99$$
$$= 0.8 \times (1.13 \times 895.99) \leftarrow \text{tax applied first}$$

Therefore, the final cost for the customer is the same regardless of whether the discount or tax is applied first.

12. a) 70%   b) $20 784.50 for needs, $12 470.70 for wants, $8313.80 for savings

13. a) Electrosource   b) Powermate

c) Answers may vary. For example, the customer should consider how many batteries are actually needed and how long any extra batteries can be stored before becoming unusable.

14. a) 12.2 cm   b) 6.1 cm   c) 36%

15. a) 22.9% increase   b) 125% increase   c) 200% increase

16. 1 minute and 35 seconds

17. a) 400%

    b) Let x be the side length of the square. Therefore, the area of the square is $(x)(x) = x^2$.

       If the side length is doubled, it becomes $2x$, and the area becomes $(2x)(2x) = 4x^2$.

       Finally, $\dfrac{4x^2}{x^2} = 4 = 400\%$.

18. a) 2700%

    b) Let x be the edge length of the cube. Therefore, the volume of the cube is $(x)(x)(x) = x^3$.

       If the edge length is tripled, it becomes $3x$, and the volume becomes $(3x)(3x)(3x) = 27x^3$.

       Finally, $\dfrac{27x^3}{x^3} = 27 = 2700\%$.