

# PROPORTIONAL RELATIONSHIPS AND CONVERSIONS

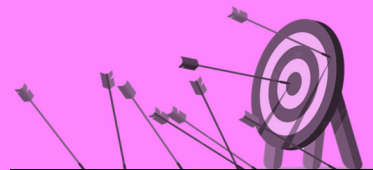


## BIG IDEAS:

- A **proportional relationship** exists between two variables if one variable is always a constant multiple of the other
- Many, but not all, **unit conversions** are examples of proportional relationships
- Not all **linear relations** are proportional

## LEARNING GOALS AND SKILL DEVELOPMENT:

You know you have met the goals for this lesson when you can:



	LEARNING GOALS	ANCHOR QUESTIONS
EMERGING	Explain what it means for a relationship between two variables to be proportional	7
	Find an unknown value in a proportional relationship	3
	Solve a proportion to determine an unknown whole number value	5

SKILL BUILDING QUESTIONS			
1	2	3	4
5	6	7	

	LEARNING GOALS	ANCHOR QUESTIONS
EVOLVING	Solve a proportion to determine an unknown decimal value	8
	Solve real-world problems by writing and solving a proportion equation	9, 13, 14
	Determine if a linear relationship is proportional or non-proportional	20

SKILL BUILDING QUESTIONS			
8	9	10	11
12	13	14	15
16	17	18	19
20			

	LEARNING GOALS	ANCHOR QUESTIONS
EXTENDING	Determine if a linear relationship is an example of direct variation or partial variation	22
	Use proportions to solve problems involving geometric figures	21, 24

SKILL BUILDING QUESTIONS			
21	22	23	24

# BUILD YOUR SKILLS

1. Imani's resting heart rate is 76 beats per minute (bpm). At this rate,

- a) how many times will her heart beat in 3 minutes?
- b) how many times will her heart beat in 1 hour?
- c) how many times will her heart beat in 30 seconds?



2. Yanush can type 110 words in 2.5 minutes. Approximately how many words can he type per minute?



3. At a constant rate, a car travels 216 kilometres in 2 hours.

- a) How far will the car travel in 3 hours?
- b) How long does it take to travel 54 km?

4. By definition, one inch is equivalent to 2.54 cm. Calculate the following conversions.

- a) 5 inches to centimetres
- b) 16.5 inches to millimetres
- c) 30 cm to inches
- d) 1 metre to inches

5. Solve each proportion (find the value of  $x$ ).

a)  $\frac{3}{4} = \frac{x}{8}$     b)  $\frac{5}{3} = \frac{x}{18}$     c)  $\frac{40}{50} = \frac{x}{10}$     d)  $\frac{x}{12} = \frac{1}{4}$     e)  $\frac{28}{21} = \frac{4}{x}$     f)  $\frac{36}{x} = \frac{4}{11}$

6. The ratio of violinists to cellists in an orchestra is 8 : 3 . If the orchestra has 12 cellists, how many violinists does it have?

7. Explain what it means for a relationship between two variables to be *proportional*.

8. Solve each proportion (find the value of  $x$ ).

a)  $\frac{1}{5} = \frac{x}{12}$     b)  $\frac{15}{8} = \frac{n}{6}$     c)  $\frac{6}{11} = \frac{y}{50}$     d)  $\frac{14}{x} = \frac{3}{5}$     e)  $\frac{9}{5} = \frac{4}{p}$     f)  $\frac{t}{7} = \frac{11}{6}$

9. A construction worker uses her boot length to estimate distances. The length of her boot is approximately 28 cm.

- a) How many centimetres is a distance that equals 17 boot lengths?
- b) How many metres is a distance that measures 32 boot lengths?
- c) How many boot lengths are needed to measure a distance of 350 cm?
- d) How many boot lengths are needed to measure a distance of 7 m?



10. In cooking, a measurement of a *cup* is equivalent to 16 tablespoons or 250 mL.

- a) A jerk chicken recipe calls for  $\frac{1}{2}$  cup of lime juice. Express this volume in millilitres.
- b) A cake recipe requires 320 mL of sugar. Express this measurement in tablespoons.

11. Using the information on the right, convert the following measurements.

- a) 5 feet to inches
- b) 8.5 yards to feet
- c) 1 yard to inches
- d) 57 inches to feet
- e) 129 feet to yards
- f) 3103.2 inches to yards

1 yard = 3 feet  
1 foot = 12 inches

12. The length of a Canadian football field is 150 yards. Given that there are 3 feet in a yard, 12 inches in a foot, and 2.54 cm in an inch, determine the exact length of a Canadian football field in metres.



13. During his record-breaking race at the 2009 IAAF World Championships, Jamaican sprinter Usain Bolt's average speed was 37.58 km/h. Express this speed in metres per second.

14. Cheetahs are the world's fastest land animal and can reach speeds of up to 70 miles per hour (mph). Express this speed in feet per second, rounded to one decimal place.

*Note: 1 mile = 5280 feet*

15. A 2-stroke go-kart engine requires a mixture of gasoline and oil in the ratio 16 : 1.

- a) How much gas should be used in a solution that has 0.5 L of oil?
- b) If a container is filled with 20 L of gasoline, how many litres of oil should be added to obtain the correct mixture?
- c) How much oil, in millilitres, is used per US gallon of gas?

*Note: 1 US gal  $\approx$  3.8 L*



16. Pasta sauce is stored in a cylindrical jar that has a diameter to height ratio of 4 : 9. If the height of the jar is 18 cm, determine the volume of sauce, in millilitres, that the jar can hold.

*Note: 1 mL = 1 cm<sup>3</sup>*

17. The floorplan for a new house shows a rectangular storage room measuring 8' 10" by 7' 6" .

- a) Express the room's perimeter as a combination of feet and inches. *Note: 1 ft = 12 in*
- b) Determine the area of the floor in square feet.
- c) If the room has an 8' ceiling, find its volume in cubic metres. *Note: 1 in = 2.54 cm*

18. Not all linear relations are proportional!

- a) Is the linear relation shown in the table on the right proportional? Explain.
- b) How is a linear relation's initial value (y-intercept) related to whether or not the relationship is proportional? Explain.
- c) Determine whether each of the following linear relations is proportional.

Time (s)	Height (m)
0	0
2	6
4	12
6	18
8	24

- i)  $d = 50t$
- ii)  $V = 2 + 8t$
- iii)  $y = \frac{1}{3}x + 9$
- iv)  $T = -1.5d$

19. The relationship between temperature in degrees Celsius ( $C$ ) and degrees Fahrenheit ( $F$ ) is given by the equation  $F = \frac{9}{5}C + 32$ . Is the relationship between the  $^{\circ}\text{C}$  temperature and the  $^{\circ}\text{F}$  temperature proportional? Explain.



20. State whether each linear relation is proportional or non-proportional.

a)

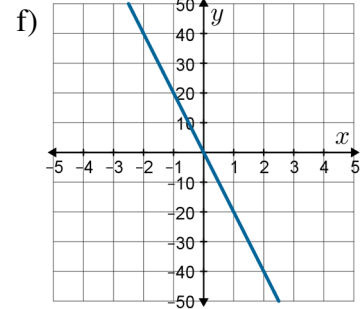
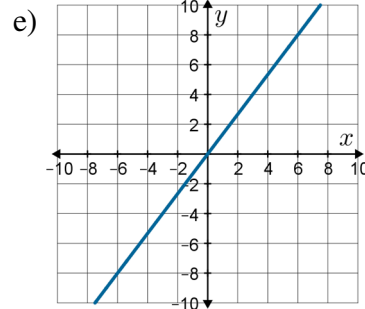
$x$	$y$
0	10
1	20
2	30
3	40
4	50

b)

$P$	$t$
-2	-8
-1	-4
0	0
1	4
2	8

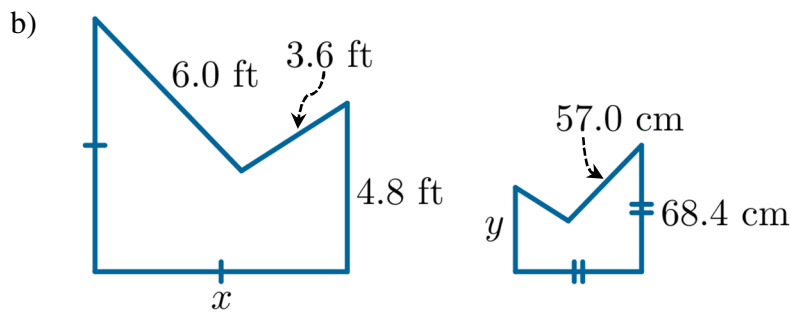
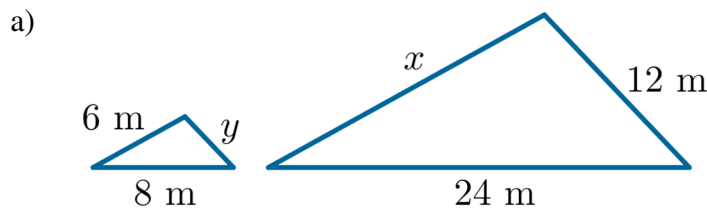
c)

$x$	$y$
-10	0
0	-1.5
10	-3
20	-4.5
30	-6



21. Two figures are said to be *similar* if they have the same shape. Specifically, similar figures have equal corresponding angles and proportional corresponding side lengths. For each of the following pairs of similar figures, determine the values of  $x$  and  $y$ .

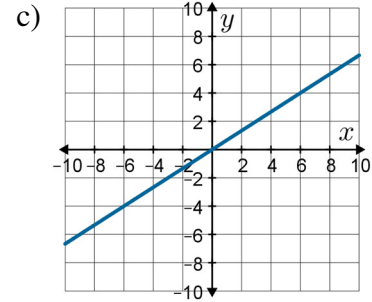
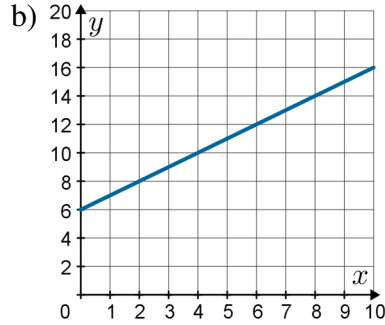
*Note: Diagram scales may not be accurately illustrated.*



22. *Direct variation* occurs when a linear relation is proportional. *Partial variation* occurs when a linear relation is not proportional. State the type of variation for each linear relation.

a)

$x$	$y$
0	0
1	16
2	32
3	48
4	64



d)  $C = 25n$     e)  $y = 8x - 15$     f)  $x + y = 4$     g)  $2x + y = 0$     h)  $x - y = 0$

23. Convert the following measurements.

a)  $1 \text{ m}^2$  to  $\text{cm}^2$     b)  $6250 \text{ mm}^2$  to  $\text{cm}^2$     c)  $0.000035 \text{ m}^3$  to  $\text{mm}^3$     d)  $3657.6 \text{ in}^2$  to  $\text{ft}^2$

24. Yassim is designing a cylindrical can. He has decided that the can will have a height of 24 cm and a radius of 6.5 cm. The material for the side of the can costs  $\$2.60/\text{m}^2$ . The material for the top and bottom costs  $\$3.10/\text{m}^2$ .

- Determine the capacity of the can to the nearest tenth of a litre. *Note:  $1 \text{ mL} = 1 \text{ cm}^3$*
- Determine the cost of the material needed for each can.

# CHECK YOUR UNDERSTANDING

- a) 228   b) 4560   c) 38
- 44
- a) 324 km   b) 0.5 hours (30 minutes)
- a) 12.7 cm   b) 419.1 mm   c) 11.8 in   d) 39.4 in
- a)  $x = 6$    b)  $x = 30$    c)  $x = 8$    d)  $x = 3$    e)  $x = 3$    f)  $x = 99$
- 32
- A relationship between two variables is proportional if one variable is a constant multiple of the other.
- a)  $x = 2.4$    b)  $n = 11.25$    c)  $y \approx 27.3$    d)  $x \approx 23.3$    e)  $p \approx 2.2$    f)  $t \approx 12.8$
- a) 476 cm   b) 8.96 m   c) 12.5   d) 25
- a) 125 mL   b) approximately 20.5 tbsp
- a) 60 in   b) 25.5 ft   c) 36 in   d) 4.75 ft   e) 43 yd   f) 86.2 yd
- 137.16 m
- 10.44 m/s
- 102.7 ft/s
- a) 8 L   b) 1.25 L   c) 237.5 mL
- 904.8 mL
- a) 32' 8"   b) 795 ft<sup>2</sup>   c) 15 m<sup>3</sup>
- a) Yes, since each height value is three times the corresponding time value.  
b) A linear relation is proportional if and only if its initial value is 0. For a proportional relationship, the dependent variable ( $y$ ) is a constant multiple of the independent variable ( $x$ ). Therefore, the initial value ( $y$ -intercept) of a proportional linear relation is a multiple of 0, which can only be 0.  
c) i) proportional   ii) not proportional   iii) not proportional   iv) proportional
- Although the relationship between temperature in degrees Celsius and degrees Fahrenheit is linear, it is not proportional since 0 °C is not equal to 0 °F, but rather 32 °F.

20. a) non-proportional    b) proportional    c) non-proportional    d) non-proportional  
e) proportional    f) proportional
21. a)  $x = 18$  m,  $y = 4$  m    b)  $x = 7.2$  ft,  $y = 45.6$  cm
22. a) direct variation    b) partial variation    c) direct variation    d) direct variation  
e) partial variation    f) partial variation    g) direct variation    h) direct variation
23. a)  $10\,000\text{ cm}^2$     b)  $62.5\text{ cm}^2$     c)  $35\,000\text{ mm}^3$     d)  $25.4\text{ ft}^2$
24. a) 3.2 L    b) \$0.34