RELATIONS OF THE FORM *xy* = *k*

BIG IDEAS:

- The relation ***xy* = *k*** consists of points for which the ***x*-value and *y*-value always multiply to *k***
- Unless $k = 0$, no points of the relation $xy = k$ have an x-coordinate or a y-coordinate of 0 (the graph touches neither the x-axis nor the y-axis)

LEARNING GOALS AND SKILL DEVELOPMENT:
You know you have met the goals for this lesson when you can:

EMERGING

LEARNING GOALS	ANCHOR QUESTIONS
Match equations of the form $xy = k$ with the corresponding graphs	3

SKILL BUILDING QUESTIONS			
1	2	3	

EVOLVING

LEARNING GOALS	ANCHOR QUESTIONS
Sketch the graph of a relation with an equation in the form $xy = k$	6
Match inequalities based on the relation $xy = k$ with the corresponding graphs	8

SKILL BUILDING QUESTIONS			
4	5	6	7
8	9	10	

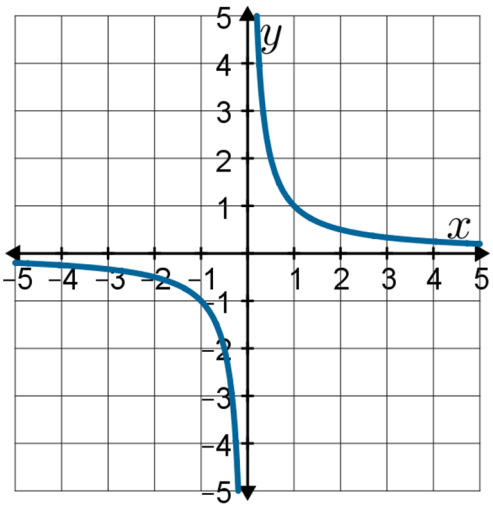
EXTENDING

LEARNING GOALS	ANCHOR QUESTIONS
Explain the relationship between the equations $xy = k$ and $y = k/x$ and make assertions about the behaviour of the corresponding graphs	11

SKILL BUILDING QUESTIONS			
11	12	13	

BUILD YOUR SKILLS

1. Consider the graph of the relation $xy = 1$ shown below.



a) Explain why the graph passes through the points $(-1,-1)$ and $\left(2,\frac{1}{2}\right)$.

b) Explain why the graph does not pass through any point with an x-value or y-value of 0.

c) Explain why the graph appears in only the top right and bottom left quadrants.

d) Explain why the graph approaches the x-axis on the right-hand side.

e) Explain why the graph's y-values increase as the x-value approaches 0 from the right.

2. Consider the relation $xy = k$, where k is a constant.

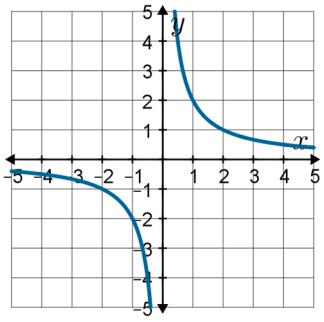
a) If $k = 4$, what must be true about the signs of x and y? Explain.

b) If $k = -8$, what must be true about the signs of x and y? Explain.

c) If $k = 0$, what can we conclude about x and y? Explain.

3. Match each graph with its corresponding equation.

a)



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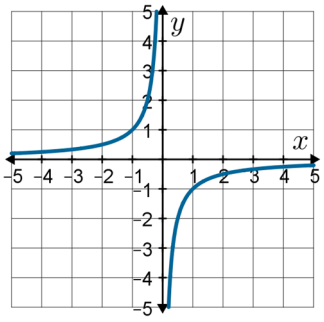

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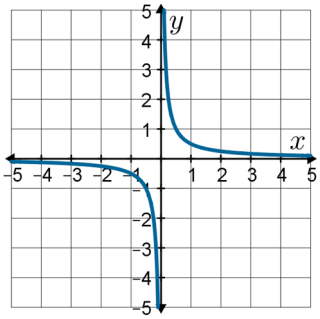
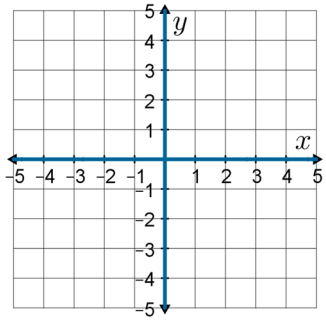


d)



i) $xy = -1$
ii) $xy = 0$
iii) $xy = 2$
iv) $xy = \frac{1}{2}$

4. Does the point $(2,4)$ satisfy the relation $xy=-8$? Explain.

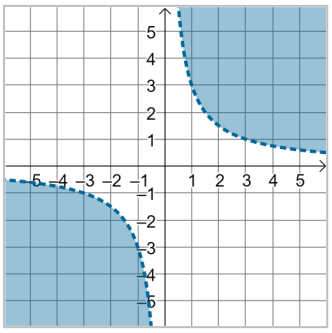
5. Does the point $(4,-7)$ satisfy the relation $xy=-28$? Explain.

6. Sketch the graph of the relation $xy=4$.

7. Sketch the graph of the relation $xy>0$. How would the appearance of the graph be different if the relation was $xy\leq 0$?

8. Match the graph of each inequality with its corresponding algebraic representation.

a)



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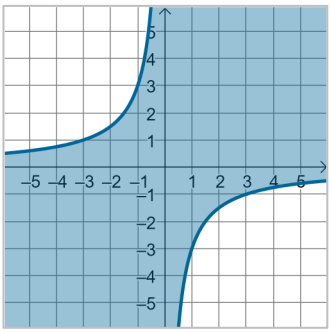
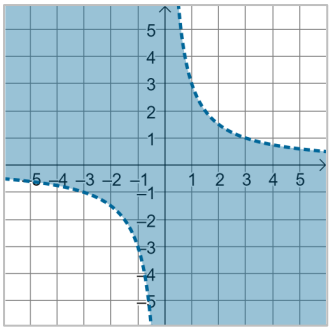


i) $xy\leq -3$
ii) $xy<3$
iii) $xy\geq -3$
iv) $xy>3$

c)



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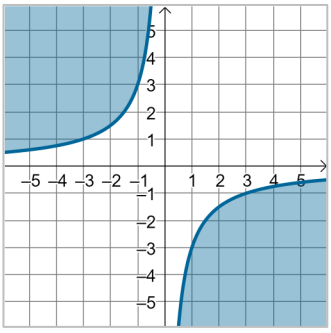
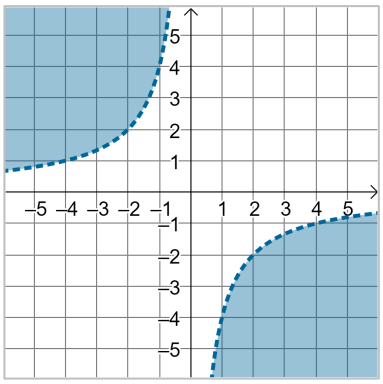


9. Sketch the graph of each relation.

a) $xy=-2$ b) $xy>1$ c) $xy\leq 3$ d) $xy\geq -1$

10. Determine an algebraic representation of the graph on the right.



11. By dividing both sides of the equation by *x*, we can express the relation $xy = 1$ as $y = \frac{1}{x}$. Using this representation of the relation,

a) explain why we cannot use an *x*-value of 0.

b) explain why the graph of the relation will not have any points with a *y*-value of 0.

c) explain what happens to the value of *y* as *x* approaches infinity.

d) explain what happens to the value of *y* as *x* approaches negative infinity.

e) explain what happens to the value of *y* as *x* approaches 0.

12. When the graph of a relation contains a break, we say that the relation is *discontinuous* at the *x*-value where the break occurs. Does the relation $xy = 2$ have any discontinuities? If so, where do they occur and for what reason do they exist?

13. An *asymptote* is a straight line (or other curve) to which a graph gradually comes closer and closer, but does not cross. State the equations of the vertical and horizontal asymptotes for the relation $xy = 1$.

CHECK YOUR UNDERSTANDING

1. a) For each point, multiplying the x-value by the y-value gives a result of 1. That is, $(-1)(-1)=1$ and $(2)\left(\frac{1}{2}\right)=1$.

b) If either x or y is 0, the the product xy is equal to 0, not 1.

c) The equation $xy=1$ tells us that x and y must multiply to a positive result $(+1)$. Therefore, x and y must be either both positive (top right quadrant) or both negative (bottom left quadrant).

d) As the x-value approaches infinity, the corresponding y-value must become closer to 0 in order for the product xy to work out to 1.

e) As the x-value approaches zero from the right, the corresponding y-value must become larger in order for the product xy to work out to 1.

2. a) x and y must have the same sign since their product is positive $(+4)$.

b) x and y must have opposite signs since their product is negative (-8).

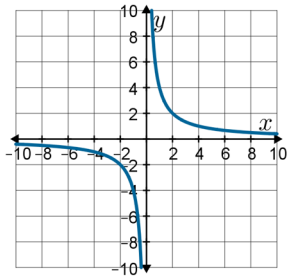
c) At least one of x and y must be 0 since their product is 0.

3. a) iii b) i c) iv d) ii

4. No, since $(2)(4)\neq -8$.

5. Yes, since $(4)(-7)=-28$.

6.



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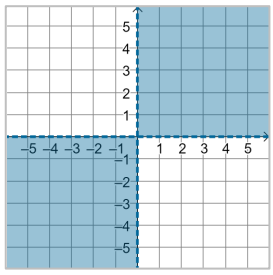
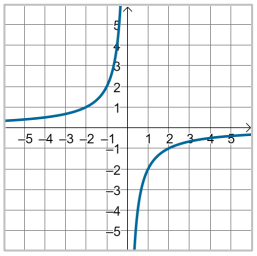


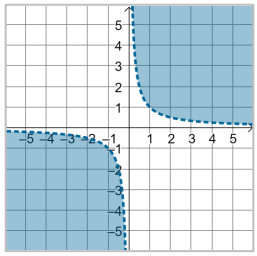
If the relation was $xy\leq 0$, the lines on the axes would be solid instead of dashed and the shading would appear in the top left and bottom right quadrants.

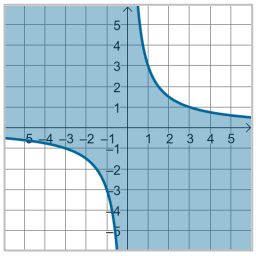
8. a) iv b) iii c) ii d) i

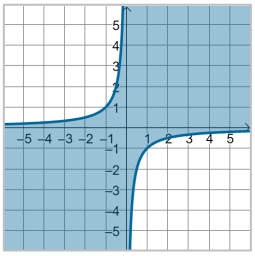
ANSWERS

ANSWERS

9. a) 

b) 

c) 

d) 

10. $xy < -4$

11. a) Division by 0 is undefined.

b) 1 divided by another value will never give a result of 0.

c) y approaches 0 (from above) since it is the result of dividing 1 by an increasingly large positive value.

d) y approaches 0 (from below) since it is the result of dividing 1 by a negative number of increasing magnitude.

e) y approaches infinity (when x approaches 0 from the right) or negative infinity (when x approaches 0 from the left) since it is the result of dividing 1 by a value of decreasing magnitude.

12. The relation has a discontinuity at $x = 0$ since there is no value of y that would give a result of 2 when multiplied by an x-value of 0.

13. The vertical asymptote is the line $x = 0$ and the horizontal asymptote is the line $y = 0$.