POWERS OF POWERS, PRODUCTS AND QUOTIENTS

BIG IDEAS:

- A **power of a power** can be simplified by **multiplying the exponents** and **keeping the base the same**
- An exponent on a **product** can be applied to **each factor** of the product
- An exponent on a **fraction** can be applied to the **numerator and denominator**

LEARNING GOALS AND SKILL DEVELOPMENT:
You know you have met the goals for this lesson when you can:

EMERGING

LEARNING GOALS	ANCHOR QUESTIONS
Identify the resulting exponent when a power is raised to a power	2, 3
Identify equivalent expressions involving powers	4

SKILL BUILDING QUESTIONS			
1	2	3	4

EVOLVING

LEARNING GOALS	ANCHOR QUESTIONS
Simplify expressions involving powers of powers	5
Simplify expressions involving powers of powers and then evaluate the result for a given value of the variable	7
Express a power using a different base	8, 9, 10

SKILL BUILDING QUESTIONS			
5	6	7	8
9	10	11	12
13			

EXTENDING

LEARNING GOALS	ANCHOR QUESTIONS
Simplify expressions involving powers of powers with negative exponents	14
Simplify multi-step expressions involving powers of powers with negative exponents	15, 16, 17

SKILL BUILDING QUESTIONS			
14	15	16	17
18	19	20	21

BUILD YOUR SKILLS

1. Show why each of the following is true.

a) $\left(2^3\right)^4 = 2^{12}$ b) $(2\times 3)^4 = 2^4\times 3^4$ c) $\left(\dfrac{2}{3}\right)^4 = \dfrac{2^4}{3^4}$

2. State the value that should be placed in each box.

a) $\left(2^3\right)^5 = 2^{\square}$ b) $\left(x^5\right)^3 = x^{\square}$ c) $\left(3^4\right)^6 = \square^{24}$ d) $\left(y^3\right)^{\square} = y^{18}$

e) $(xy)^4 = x^4y^{\square}$ f) $(3pq)^{\square} = 3^{10}p^{10}q^{10}$ g) $\left(\dfrac{5}{6}\right)^7 = \dfrac{5^7}{6^{\square}}$ h) $\left(\dfrac{a}{b}\right)^{\square} = \dfrac{a^9}{b^9}$

3. State the value that should be placed in each box.

a) $\left(x^4y^6\right)^2 = x^{\square}y^{12}$ b) $\left(35x^4y^6\right)^2 = 1225x^{\square}y^{12}$ c) $\left(\dfrac{x^8}{y^3}\right)^4 = \dfrac{x^{\square}}{y^{12}}$ d) $\left(\dfrac{a^2}{b^3}\right)^{\square} = \dfrac{a^{14}}{b^{21}}$

4. The table on the right contains five expressions. Identify all of these expressions that are equivalent to $\left(p^3\right)^2$.

$p\times p\times p\times p\times p\times p$
$p^2\times p^2\times p^2$
$p^3\times p^3$
p^5
p^6

5. Use at least one exponent rule to find an equivalent/simplified expression.

a) $\left(6^5\right)^4$ b) $\left(x^3\right)^6$ c) $(xy)^8$ d) $\left(\dfrac{x}{y}\right)^7$ e) $(2x)^3$ f) $\left(\dfrac{1}{2}xy\right)^2$

g) $\left(\dfrac{a}{4}\right)^3$ h) $\left(x^2y^3\right)^5$ i) $\left(\dfrac{x^8}{y^4}\right)^3$ j) $\left(4^2a^3b^6\right)^3$ k) $\left(-2mn^4\right)^6$ l) $\left(\dfrac{-5}{p^8}\right)^3$

6. State whether the two expressions are equivalent or not equivalent.

a) $\left(x^6\right)^2$ and $\left(x^3\right)^4$ b) $\left(x^2y^4\right)^4$ and $\left(xy^2\right)^8$ c) $\left(\dfrac{m^{10}}{n^{14}}\right)^4$ and $\left(\dfrac{n^5}{m^7}\right)^8$

7. Use at least one exponent rule to find an equivalent/simplified expression and then evaluate for $x=-1$ and $y=2$.

a) $\left(\dfrac{x}{y}\right)^3$ b) $(3xy)^4$ c) $(x^2y)^2$ d) $\left(\dfrac{x^5}{y^3}\right)^2$ e) $\left(\dfrac{4}{y^2}\right)^3$ f) $\left(\dfrac{1}{2}x^5y^2\right)^4$

8. Express 4^3 as a power with a base of 2.

9. Express 25^4 as a power with a base of 5.

10. Express 27^2 as a power with a base of 3.

11. Express $\left(\dfrac{1}{81}\right)^4$ as a power with a base of $\dfrac{1}{3}$.

12. Without actually calculating the value of either power, show that 16^3 is equal to 4^6.

13. Without actually calculating the value of either power, show that 5^{183} is equal to 125^{61}.

14. Write as a single power and then evaluate. Express all answers in exact form.

a) $(2^2)^3$ b) $(3^2)^{-2}$ c) $(4^{-1})^{-3}$ d) $\left[(-2)^2\right]^{-4}$

15. Simplify $\left[(x^2)^3\right]^4$.

16. Simplify $\left(\left((m^2n^3)^2\right)^4\right)^5$.

17. Simplify $\left[\dfrac{(a^3)^2}{(b^2)^4}\right]^5$.

18. Kendra needs to quickly determine whether 3^{40} is greater than or less than 4^{30}, but she does not have access to a calculator. How can she use her knowledge that $3^4 = 81$ and $4^3 = 64$ to solve her problem. Which power has the greater value?

19. A cube has a side length of x^4 cm.

a) Express the volume of the cube as a power of a power.

b) Express the volume of the cube in simplified form.

c) Determine an expression for the surface area of the cube in simplified form.

20. Simplify. Express all answers using positive exponents.

a) $\left(a^5\right)^{-2}$ b) $\left(x^{-3}\right)^{-4}$ c) $\left(x^{-2}y^{-3}\right)^{-4}$ d) $\left(a^{-4}b^3\right)^{-2}$ e) $\left(3p^{-4}q^2\right)^4$ f) $\left(-2xy^{-3}\right)^{-6}$

21. Simplify. Express all answers using positive exponents.

a) $\left(\dfrac{x^5}{y^{-1}}\right)^3$ b) $\left(\dfrac{a^{-4}}{b^{-3}}\right)^2$ c) $\left(\dfrac{m^{-5}}{n^4}\right)^3$ d) $\left(\dfrac{x^{-4}}{y^{-2}}\right)^{-3}$

CHECK YOUR UNDERSTANDING

1. a) One possible explanation is as follows:

$$\begin{aligned}(2^3)^4 &= 2^3\times2^3\times2^3\times2^3\\ &=(2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)\\ &=2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\\ &=2^{12}\end{aligned}$$

b) One possible explanation is as follows:

$$\begin{aligned}(2\times3)^4 &= (2\times3)\times(2\times3)\times(2\times3)\times(2\times3)\\ &=2\times3\times2\times3\times2\times3\times2\times3\\ &=2\times2\times2\times2\times3\times3\times3\times3\\ &=(2\times2\times2\times2)\times(3\times3\times3\times3)\\ &=2^4\times3^4\end{aligned}$$

c) One possible explanation is as follows:

$$\begin{aligned}\left(\frac{2}{3}\right)^4 &= \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\\ &=\frac{2\times2\times2\times2}{3\times3\times3\times3}\\ &=\frac{2^4}{3^4}\end{aligned}$$

2. a) 15 b) 15 c) 3 d) 6 e) 4 f) 10 g) 7 h) 9

3. a) 8 b) 8 c) 32 d) 7

4. $p\times p\times p\times p\times p\times p$, $p^2\times p^2\times p^2$, $p^3\times p^3$, p^6

5. a) 6^{20} b) x^{18} c) x^8y^8 d) $\frac{x^7}{y^7}$ e) $8x^3$ f) $\frac{1}{4}x^2y^2$ g) $\frac{a^3}{64}$ h) $x^{10}y^{15}$ i) $\frac{x^{24}}{y^{12}}$

j) $4096a^9b^{18}$ k) $64m^6n^{24}$ l) $-\frac{125}{p^{24}}$

6. a) equivalent b) equivalent c) not equivalent

7. a) $\frac{x^3}{y^3}$; $-\frac{1}{8}$ b) $81x^4y^4$; 1296 c) x^4y^2 ; 4 d) $\frac{x^{10}}{y^6}$; $\frac{1}{64}$ e) $\frac{64}{y^6}$; 1

f) $\frac{1}{16}x^{20}y^8$; 16

ANSWERS

8. 2^6

9. 5^8

10. 3^6

11. $\left(\frac{1}{3}\right)^{16}$

12. $16^3 = \left(4^2\right)^3$

$= 4^6$

13. $5^{183} = 5^{3\times 61}$

$= \left(5^3\right)^{61}$

$= 125^{61}$

14. a) 2^6 ; 64 b) 3^{-4} ; $\frac{1}{81}$ c) 4^3 ; 64 d) $(-2)^{-8}$; $\frac{1}{256}$

15. x^{24}

16. $m^{80}n^{120}$

17. $\frac{a^{30}}{b^{40}}$

18. 3^{40} can be expressed as $\left(3^4\right)^{10}$. 4^{30} can be expressed as $\left(4^3\right)^{10}$. Since Kendra knows that $3^4 > 4^3$, she can conclude that $\left(3^4\right)^{10} > \left(4^3\right)^{10}$. Therefore, 3^{40} is greater than 4^{30}.

19. a) $\left(x^4\right)^3$ cm^3 b) x^{12} cm^3 c) $6x^8$ cm^2

20. a) $\frac{1}{a^{10}}$ b) x^{12} c) x^8y^{12} d) $\frac{a^8}{b^6}$ e) $\frac{81q^8}{p^{16}}$ f) $\frac{y^{18}}{64x^6}$

21. a) $x^{15}y^3$ b) $\frac{b^6}{a^8}$ c) $\frac{1}{m^{15}n^{12}}$ d) $\frac{x^{12}}{y^6}$