

MULTIPLYING POLYNOMIALS BY MONOMIALS (THE DISTRIBUTIVE PROPERTY)

CHECK YOUR UNDERSTANDING

PART A

1) Explain why $2(3+4)$ is equal to $2(3)+2(4)$.

2) State the value that should be placed in each box.

a) $\square(3+7) = 5(3)+5(7)$

b) $\square(18-3) = (-2)(18) - (-2)(3)$

c) $\square(x+6) = 8x+8(6)$

d) $\square(7x-9) = 3(7x)-3(9)$

e) $\square(x+4) = x^2+4x$

f) $\square(x^2-2) = -4x^2+8$

3) Expand.

a) $2(x+6)$

b) $3(x-4)$

c) $-4(m+2)$

d) $-3(y-7)$

e) $3(2x+5)$

f) $-5(2a+9)$

g) $-6(-2x+3)$

h) $-3(-3p-2)$

4) Expand.

a) $2(x^2+6x)$

b) $-3(x^2+8x+5)$

c) $y(y+9)$

d) $-1(m+6)$

e) $-(5x-7)$

f) $2x(3x+4)$

g) $-3k(5k-6)$

h) $-2t(-t-4)$

PART B

5) When asked to simplify the expression $8(3x+4)$, Natasha stated, "I have to follow BEDMAS, so I must first add the terms in the brackets." Explain why Natasha's approach would not be effective here.

6) Expand and simplify.

a) $5(x-6)$

b) $-4(9k-8)$

c) $8(6+4b)$

d) $-3(-2x-7)$

e) $(5p+8)(-3)$

f) $x(5x+4)$

g) $(-2x)(6x-7)$

h) $z(-8z-7)$

i) $-(4x^2+9x-8)$

j) $(7-10p)\times 4p$

k) $-x(-8-6x)$

l) $-2x(x^2+4x+6)$

7) Expand, simplify and then evaluate for $x=3$ and $y=-2$.

a) $5(x+3)$

b) $2y(y-7)$

c) $(6-x)(-3x)$

d) $-y(2y+3)$

e) $4x(2y+3)$

8) An emergency roofing repair company charges \$120 per visit plus \$75 per hour.

a) Determine an expression to represent the total cost of a repair visit that requires t hours of work.

b) On weekends and holidays, the total cost is doubled. Determine a simplified expression for the total cost of a weekend repair visit that takes t hours.

c) Use your expression from part (b) to determine the total cost of a weekend repair visit that required 1 hour and 30 minutes.



9) Simplify.

- a) $5(x+9)+12x$ b) $m+3(2m-4)$ c) $x+5-6(3x+2)$
 d) $2(x+7)+3(x+4)$ e) $-5(2x+3)+(6x-4)$ f) $-6(2y+5)+3(5y-1)$
 g) $(x-4)(3)-(2x+6)$ h) $2(5x+3)-4(x+3)$ i) $-(-5m+8)-2(6m-7)+3(m-1)$

10) Simplify.

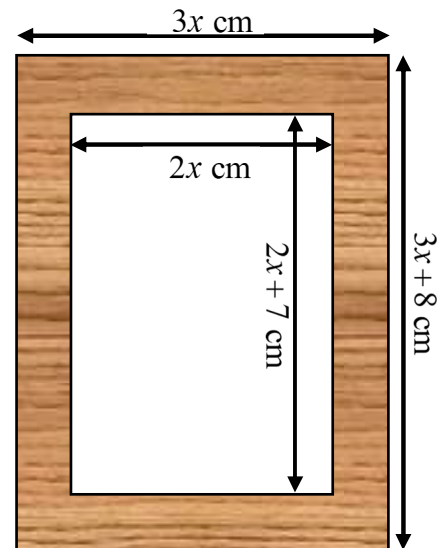
- a) $x(x+5)+2x(x-4)$ b) $2x(3x-4)-3x(5x+6)$
 c) $-3a(2a+4)-2a(3-a)$ d) $2(3y^2+4y-5)+4(y^2-3y+1)$
 e) $-5(2x^2-8x+1)-3(-7x^2+10x-12)$ f) $3(x^2+5x-3)+(7x+4)(2x)$

11) The width of a rectangle is represented by the expression $3x+5$. The rectangle's length is three times its width.

- a) Determine a simplified expression for the length of the rectangle.
 b) Determine a simplified expression that represents the perimeter of the rectangle.

12) A picture frame is constructed according to the diagram on the right.

- a) Determine a simplified expression for the area of the entire frame (picture and border).
 b) Determine a simplified expression for the area of the picture.
 c) Determine a simplified expression for the area of the border.



13) Expand and simplify.

- a) $\frac{1}{2}\left(\frac{3}{4}y-\frac{1}{3}\right)$ b) $\frac{1}{3}(3a+6)$ c) $-\frac{2}{3}(8m+6)$

14) Simplify.

- a) $\frac{1}{2}(8a-4)-\frac{2}{3}(6a+12)$ b) $\frac{5}{6}\left(\frac{1}{2}x+\frac{1}{3}\right)-\frac{2}{3}\left(\frac{3}{4}x-\frac{7}{6}\right)$ c) $\frac{1}{4}\left(\frac{1}{6}x+\frac{4}{5}\right)-\frac{5}{3}\left(\frac{3}{2}x-\frac{3}{4}\right)$

PART C

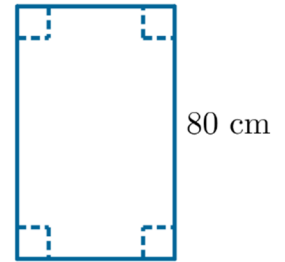
15) Expand and simplify the expression $2[-3x(5x-8)]$.

16) Use the distributive property of multiplication over addition to explain why $\frac{7x+5}{3} = \frac{7}{3}x + \frac{5}{3}$.

17) Simplify.

- a) $\frac{5x+9}{2}$ b) $\frac{8x-12}{2}$ c) $\frac{15y^2-9y+12}{3}$ d) $\frac{17x^2+10x}{x}$ e) $\frac{24x^2-16x}{2x}$
 f) $\frac{4x^3+12x^2-20x}{-4x}$ g) $\frac{6y^5-8y^4+2y^3-10y^2}{2y^2}$ h) $\frac{t^7-7t^5+14t^3}{-6t^2}$

- 18) An open-top box will be created by removing square corners from a rectangular piece of cardboard and then folding up the resulting flaps. The length of the original piece of cardboard is 80 cm. The width of the box will be three times the side length of the removed squares.



- a) If the removed squares have a side length of x cm, determine a simplified expression to represent the area of the bottom of the box.
 b) Determine a simplified expression to represent the volume of the box.

- 19) Expand and simplify.

a) $2[x+3(2x-4)]$ b) $-3[2(3y+5)-3(4y)]$ c) $-[6x(2x+5)-3x(x-3)+4(x+7)]$

- 20) To multiply the binomial $x+2$ by the binomial $x+3$, we can use the distributive property several times, as shown below.

$$\begin{aligned}(x+2)(x+3) &= (x+2)(x) + (x+2)(3) \\ &= x^2 + 2x + 3x + 6 \\ &= x^2 + 5x + 6\end{aligned}$$

- a) Expand and simplify.

- i) $(x+5)(x+4)$ ii) $(2x+3)(x+1)$ iii) $(x+4)(x-2)$ iv) $(3x-1)(2x-5)$
 b) Can you think of a shortcut for multiplying two binomials? Explain.
 c) Expand and simplify the expression $2(3y-4)(5-4y)$.
 d) Expand and simplify the expression $(n+2)(n^2-5n+3)$.

- 21) The following expression is expanded and simplified. Determine the result.

$$(x-a)(x-b)(x-c)\dots(x-z)$$

ANSWERS

- 1) $2(3+4)$ represents two groups of the sum of 3 and 4. That is, $2(3+4)$ is two groups of 7.
Two groups of 7 is equal to two groups of 3 plus two groups of 4, or $2(3)+2(4)$.
- 2) a) 5 b) -2 c) 8 d) 3 e) x f) -4
- 3) a) $2x+12$ b) $3x-12$ c) $-4m-8$ or $-4m+(-8)$ d) $-3y+21$ or $-3y-(-21)$
e) $6x+15$ f) $-10a-45$ or $-10a+(-45)$ g) $12x-18$ or $12x+(-18)$
h) $9p+6$ or $9p-(-6)$
- 4) a) $2x^2+12x$ b) $-3x^2-24x-15$ or $-3x^2+(-24)x+(-15)$ c) y^2+9y
d) $-m-6$ or $-m+(-6)$ e) $-5x+7$ or $-5x-(-7)$ f) $6x^2+8x$
g) $-15k^2+18k$ or $-15k^2-(-18)k$ h) $2t^2+8t$ or $2t^2-(-8)t$
- 5) The terms inside the brackets are not like terms and therefore cannot be combined.
- 6) a) $5x-30$ b) $-36k+32$ c) $32b+48$ d) $6x+21$ e) $-15p-24$ f) $5x^2+4x$
g) $-12x^2+14x$ h) $-8z^2-7z$ i) $-4x^2-9x+8$ j) $-40p^2+28p$ k) $6x^2+8x$
l) $-2x^3-8x^2-12x$
- 7) a) $5x+15$; 30 b) $2y^2-14y$; 36 c) $3x^2-18x$; -27 d) $-2y^2-3y$; -2
e) $8xy+12x$; -12
- 8) a) $120+75t$ b) $240+150t$ c) \$465
- 9) a) $17x+45$ b) $7m-12$ c) $-17x-7$ d) $5x+26$ e) $-4x-19$ f) $3y-33$
g) $x-18$ h) $6x-6$ i) $-4m+3$
- 10) a) $3x^2-3x$ b) $-9x^2-26x$ c) $-4a^2-18a$ d) $10y^2-4y-6$ e) $11x^2+10x+31$
f) $17x^2+23x-9$
- 11) a) $9x+15$ b) $24x+40$
- 12) a) $9x^2+24x$ cm^2 b) $4x^2+14x$ cm^2 c) $5x^2+10x$ cm^2
- 13) a) $\frac{3}{8}y-\frac{1}{6}$ b) $a+2$ c) $-\frac{16}{3}m-4$
- 14) a) -10 b) $-\frac{1}{12}x+\frac{19}{18}$ c) $-\frac{59}{24}x+\frac{29}{20}$ 15) $-30x^2+48x$
- 16) Dividing by 3 is equivalent to multiplying by $\frac{1}{3}$. Therefore,
- $$\begin{aligned}\frac{7x+5}{3} &= \frac{1}{3}(7x+5) \\ &= \frac{1}{3}(7x) + \frac{1}{3}(5) \\ &= \frac{7}{3}x + \frac{7}{5}\end{aligned}$$
- 17) a) $\frac{5}{2}x+\frac{9}{2}$ b) $4x-6$ c) $5y^2-3y+4$ d) $17x+10$ e) $12x-8$ f) $-x^2-3x+5$
g) $3y^3-4y^2+y-5$ h) $-\frac{1}{6}t^5+\frac{7}{6}t^3-\frac{7}{3}t$
- 18) a) $-6x^2+240x$ cm^2 b) $-6x^3+240x^2$ cm^3
- 19) a) $14x-24$ b) $18y-30$ c) $-9x^2-43x-28$
- 20) a) i) $x^2+9x+20$ ii) $2x^2+5x+3$ iii) x^2+2x-8 iv) $6x^2-17x+5$
b) One possibility is to multiply each term of the first binomial by each term of the second binomial and then collect like terms.
c) $-24y^2+62y-40$ d) n^3-3n^2-7n+6
- 21) 0