

MULTIPLYING AND DIVIDING POWERS WITH THE SAME BASE



BIG IDEAS:

- When two powers with the **same base** are **multiplied**, their **exponents are added** and the **base remains the same**
- When two powers with the **same base** are **divided**, their **exponents are subtracted** and the **base remains the same**

LEARNING GOALS AND SKILL DEVELOPMENT:

You know you have met the goals for this lesson when you can:



	LEARNING GOALS	ANCHOR QUESTIONS
EMERGING	Identify the resulting exponent when two powers are multiplied	3
	Identify the resulting exponent when two powers are divided	4
	Simplify expressions involving multiplication of powers	5
	Simplify expressions involving division of powers	6

SKILL BUILDING QUESTIONS			
1	2	3	4
5	6		

	LEARNING GOALS	ANCHOR QUESTIONS
EVOLVING	Simplify multi-step expressions involving multiplication and division of powers	7
	Simplify multi-step expressions involving multiplication and division of powers and then evaluate for a given value of the variable	9
	Solve real-world problems, such as determining area and volume, and express the result as a single power	16, 18

SKILL BUILDING QUESTIONS			
7	8	9	10
16	17	18	

	LEARNING GOALS	ANCHOR QUESTIONS
EXTENDING	Simplify expressions involving multiplication of powers with negative exponents	11
	Simplify expressions involving division of powers with negative exponents	12
	Simplify multi-step expressions involving multiplication and division of powers with negative exponents	15

SKILL BUILDING QUESTIONS			
11	12	13	14
15	19		

BUILD YOUR SKILLS

1. Show why $2^4 \times 2^3 = 2^7$.

2. Show why $\frac{2^8}{2^5} = 2^3$.

3. State the value that should be placed in each box.

a) $2^3 \times 2^4 = 2^{\square}$

b) $(x^5)(x^2) = x^{\square}$

c) $5(5^{11}) = 5^{\square}$

d) $(10^6)(10^2)(10) = 10^{\square}$

e) $(-6)^7(-6)^8 = \square^{15}$

f) $x^{14}x^{\square} = x^{26}$

4. State the value that should be placed in each box.

a) $3^9 \div 3^4 = 3^{\square}$

b) $\frac{a^{12}}{a^{10}} = a^{\square}$

c) $\frac{y^8}{y} = y^{\square}$

d) $\frac{6^{15}}{6^{11}} = \square^4$

e) $\frac{2^{18}}{\square^5} = 2^{13}$

5. Express each of the following as a single power.

a) $5^2 \times 5^{10}$

b) $(2.4^3)(2.4^8)$

c) $(x^{15})(x^3)$

d) $1.5(1.5^{12})$

e) $m^3(m^6)$

f) $\left(\frac{2}{3}\right)^4 \left(\frac{2}{3}\right)^6$

g) $(7^5)(7^3)(7^4)$

h) $a(a^9)(a^2)$

i) $\left(\frac{1}{6}\right)^2 \left(\frac{1}{6}\right)^5 \left(\frac{1}{6}\right)^7 \left(\frac{1}{6}\right)^3$

6. Express each of the following as a single power.

a) $15^{14} \div 15^6$

b) $\frac{(-8)^{10}}{(-8)^3}$

c) $\frac{b^{13}}{b^4}$

d) $\left(\frac{3}{7}\right)^{10} \div \left(\frac{3}{7}\right)^4$

e) $\frac{3.78^9}{3.78^5}$

f) $\frac{\left(\frac{1}{3}\right)^7}{\left(\frac{1}{3}\right)^3}$

7. Simplify.

a) $\frac{(5^8)(5^9)}{5^7}$ b) $\frac{x^8(x^{10})}{x^4}$ c) $\frac{\left(\frac{5}{6}\right)^{17}}{\left(\frac{5}{6}\right)^8\left(\frac{5}{6}\right)^4}$ d) $\frac{a^5(a^7)}{a(a^6)}$ e) $\frac{y^4y^5}{y^6y^2}$ f) $x^2\left(\frac{x^{11}}{x^5}\right)$

g) $\frac{(-6)^{12}}{-6(-6)^2(-6)^3}$ h) $\frac{\left(\frac{7}{8}\right)^{15} \div \left(\frac{7}{8}\right)^8}{\left(\frac{7}{8}\right)^2\left(\frac{7}{8}\right)^3}$ i) $\left(\frac{x^{20}}{x^{14}}\right)\left(\frac{x^{18}}{x^{15}}\right)$ j) $\frac{4.2^{13}}{4.2^5} \div \frac{4.2^8}{4.2^3}$

8. Simplify and evaluate.

a) $\frac{7^8}{7^6}$ b) $\frac{(3^6)(3^5)}{3^8}$ c) $\frac{(4^{10})(4^2)}{(4^4)(4^5)}$ d) $\frac{5.8^{11}}{(5.8^6)(5.8^4)}$ e) $\frac{\left(\frac{2}{3}\right)^2\left(\frac{2}{3}\right)^5}{\left(\frac{2}{3}\right)^4\left(\frac{2}{3}\right)}$

9. Simplify and then evaluate for $x = 2$ and $y = 3$.

a) $(x)(x^2)(x^3)$ b) $\frac{(x^5)(x^4)(x^{10})}{(x^6)(x^8)}$ c) $y^2\left(\frac{y^8}{y^7}\right)$ d) $\left(\frac{y^6}{y^4}\right)\left(\frac{y^9}{y^7}\right)$ e) $\frac{\left(\frac{x^{19}}{x^8}\right)}{\left(\frac{x^6}{x^2}\right)}$

10. The product of two powers is 5^{12} . The quotient of the same two powers is 5^6 . Find the two powers.

11. Write as a single power and then evaluate. Express all answers in exact form.

a) $2^5 \times 2^{-3}$ b) $(3^{-4})(3^7)$ c) $4(4^{-1})$ d) $5^6(5^{-8})$
e) $(2^{-4})(2^{-3})$ f) $(-4)^{-5}(-4)^7$ g) $(-5)^3(-5)^{-6}$ h) $(10)^{-5}(10)^{12}(10)^{-4}$

12. Write as a single power and then evaluate. Express all answers in exact form.

a) $\frac{4^{10}}{4^{12}}$ b) $\frac{2^2}{2^{-3}}$ c) $\frac{2^{-2}}{2^3}$ d) $\frac{3^{-1}}{3^{-4}}$ e) $\frac{5^{-12}}{5^{-9}}$

13. Why must we be cautious when stating that the expressions $\frac{x^5}{x^2}$ and x^3 are equivalent?

14. State whether the two expressions are equivalent or not equivalent.

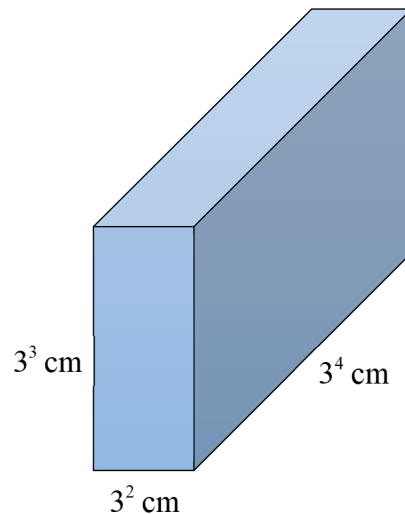
a) x^2x^3 and x^6 b) $\frac{y^{12}}{y^6}$ and y^2 , for $y \neq 0$ c) $\frac{z}{z^8}$ and $\frac{1}{z^7}$ d) $\frac{m^5m^{-1}}{m^3}$ and m , for $m \neq 0$

15. Simplify. Express all answers using positive exponents.

a) $(a^{-3})(a^5)$ b) x^5x^{-7} c) $y^{-10}y^{-2}$ d) $\frac{a^3}{a^7}$ e) $\frac{c^4}{c^{-3}}$

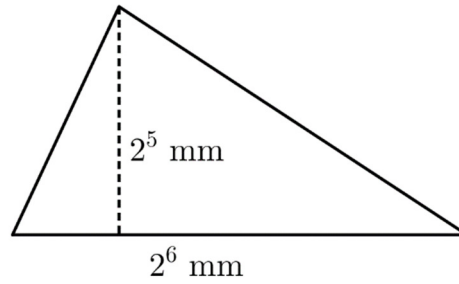
f) $\frac{x^{-15}}{x^{-9}}$ g) $\frac{y^{-9}}{y^{-12}}$ h) $\frac{d^{-6}}{d^5}$ i) $\frac{(a^{-13})(a^{15})}{(a)(a^4)}$ j) $\left(\frac{x}{x^7}\right)\left(\frac{x^{20}}{x^{12}}\right)$

16. Determine the volume of the rectangular prism shown below. Express your answer as a power.



17. A container has a volume of 5^6 cm³. A smaller container has a volume of 5^4 cm³. How many times larger is the volume of the bigger container than that of the smaller container?

18. Determine the area of the triangle shown below. Express your answer as a power.



19. We can use an exponent law to show why $a^0 = 1$ (in most cases).

- Determine the value of $\frac{2^3}{2^3}$ by first calculating the value of 2^3 and then dividing.
- Evaluate $\frac{2^3}{2^3}$ using an exponent law. Express your answer as a power.
- Based on your results from parts (a) and (b), what is the value of 2^0 ?
- Can you think of any cases where this reasoning would fail? Explain.

CHECK YOUR UNDERSTANDING

1. One possible explanation is as follows:

$$\begin{aligned} 2^4 \times 2^3 &= (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2) \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^7 \end{aligned}$$

2. One possible explanation is as follows:

$$\begin{aligned} \frac{2^8}{2^5} &= \frac{2 \times 2 \times 2 \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}} \\ &= 2 \times 2 \times 2 \\ &= 2^3 \end{aligned}$$

3. a) 7 b) 7 c) 12 d) 9 e) -6 f) 12

4. a) 5 b) 2 c) 7 d) 6 e) 2

5. a) 5^{12} b) 2.4^{11} c) x^{18} d) 1.5^{13} e) m^9 f) $\left(\frac{2}{3}\right)^{10}$ g) 7^{12} h) a^{12} i) $\left(\frac{1}{6}\right)^{17}$

6. a) 15^8 b) $(-8)^7$ c) b^9 d) $\left(\frac{3}{7}\right)^6$ e) 3.78^4 f) $\left(\frac{1}{3}\right)^4$

7. a) 5^{10} b) x^{14} c) $\left(\frac{5}{6}\right)^5$ d) a^5 e) y

f) x^8 g) $(-6)^6$ h) $\left(\frac{7}{8}\right)^2$ i) x^9 j) 4.2^3

8. a) 7^2 ; 49 b) 3^3 ; 27 c) 4^3 ; 64 d) 5.8^1 ; 5.8 e) $\left(\frac{2}{3}\right)^2$; $\frac{4}{9}$

9. a) x^6 ; 64 b) x^5 ; 32 c) y^3 ; 27 d) y^4 ; 81 e) x^7 ; 128

10. 5^9 and 5^3

11. a) 2^2 ; 4 b) 3^3 ; 27 c) 4^0 ; 1 d) 5^{-2} ; $\frac{1}{25}$ e) 2^{-7} ; $\frac{1}{128}$ f) $(-4)^2$; 16

g) $(-5)^{-3}$; $-\frac{1}{125}$ h) 10^3 ; 1000

12. a) $4^{-2}; \frac{1}{16}$ b) $2^5; 32$ c) $2^{-5}; \frac{1}{32}$ d) $3^3; 27$ e) $5^{-3}; \frac{1}{125}$

13. If $x = 0$, the expression $\frac{x^5}{x^2}$ is undefined (due to division by 0), whereas the expression x^3 is not. The two expressions are equivalent for all real number values of x except 0.

14. a) not equivalent b) not equivalent c) equivalent d) equivalent

15. a) a^2 b) $\frac{1}{x^2}$ c) $\frac{1}{y^{12}}$ d) $\frac{1}{a^4}$ e) c^7

f) $\frac{1}{x^6}$ g) y^3 h) $\frac{1}{d^{11}}$ i) $\frac{1}{a^3}$ j) x^2

16. 3^9 cm^3

17. 25 times larger

18. 2^{10} mm^2

19. a) 1 b) 2^0 c) 1

d) Using this reasoning with a power that has a base of zero would fail, since it would lead to division by zero, which is not defined.