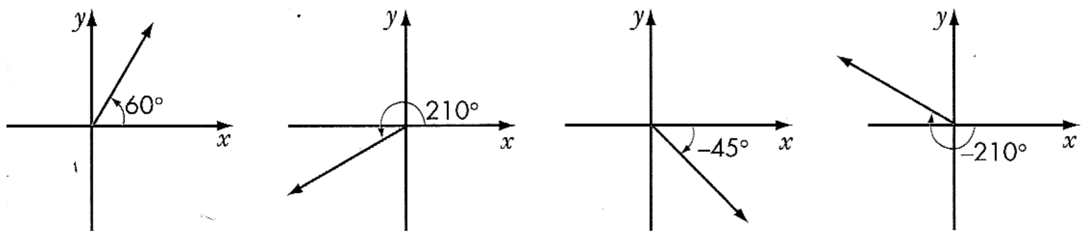
RADIAN MEASURE

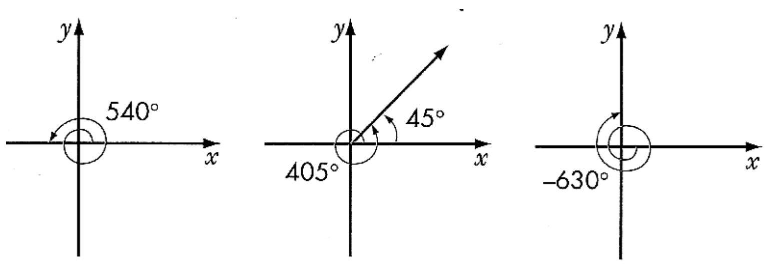
Positive, Negative and Coterminal, Oh My!

In the past, all of the angles you worked with had measures ranging from 0° to 360°. There are actually many other possible measures…the secret's out!

The angles we have seen in the past have all been positive, and were created by rotating the terminal arm counterclockwise from the positive *x*-axis. If we rotate clockwise, however, we'll get a negative angle. The following diagrams sum it up…



It's also possible for the terminal arm to rotate past 360° (or -360°).



When two angles in standard position have the same terminal arm, they are called **coterminal angles**.

For example, as shown in the diagram on the left, 45° and 405° are coterminal angles.

Notice that coterminal angles differ by a multiple of 360°.

Recall that the **related acute angle** for an angle in standard position is the **acute** angle between the terminal arm and the ***x*-axis**.

Example

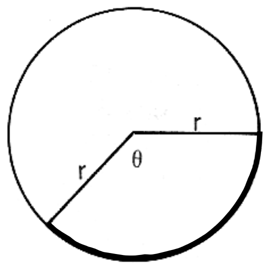
For each of the following angles, draw a diagram of the angle in standard position and find the related acute angle.

a) 150° b) -110° c) 73° d) 470°

Introducing…Radian Measure

Whenever we have referred to angles in the past, we have always used degrees to measure them. There is, however, another type of angle measurement, known as **radian measure**, which is commonly used in mathematics and science.

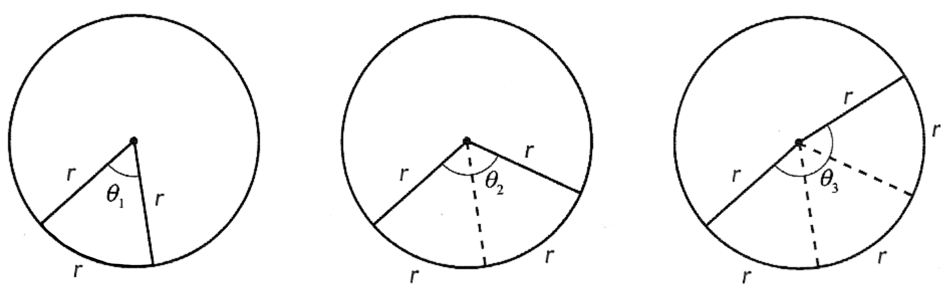
Consider the following diagram, in which the angle θ is formed by two radii of the circle.



Notice that as the angle θ increases, the length of the arc carved out on the circle also increases. When the length of the arc is exactly the same as the length of the circle's radius, the measure of angle θ is one **radian**.

One **radian** is the measure of an angle which is subtended at the centre of a circle by an arc equal in length to the radius of the circle.

A few diagrams to demonstrate…



$\theta_1 = 1$ rad $\qquad \theta_2 = 2$ rad $\qquad \theta_3 = 3$ rad

The diagrams above lead to the following generalization:

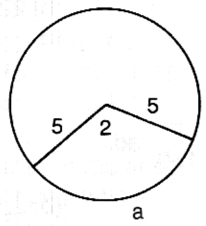
$$\text{number of radians} = \frac{\text{arc length}}{\text{radius}} \quad \text{or} \quad \theta = \frac{a}{r}$$

Does this relationship make sense? If not, look at the previous diagrams again.

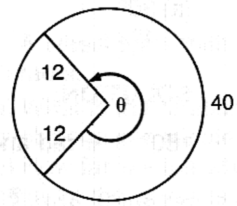
By the way, when working in radians, the unit *rad* is often omitted. For example, an angle measure written as 2 is assumed to be 2 rad, and an angle written as π is π rad.

Examples
Find the indicated quantity in each of the following.

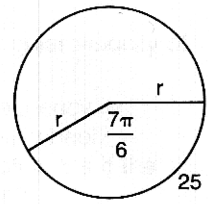
(a)



(b)



(c)



How do we convert between degrees and radians?

Well, for any circle with a radius r, the circumference is given by $2\pi r$. So, in radian measure, the angle created by one complete revolution is

$$\frac{\text{arc length}}{\text{radius}} = \frac{2\pi r}{r}$$
$$= 2\pi \text{ rad}$$

We also know that one full revolution is 360°.

$$\therefore 2\pi \text{ rad} = 360°$$
$$\pi \text{ rad} = 180°$$

The above relationship is very important and deserves repeating…

$$\boldsymbol{\pi \text{ rad} = 180°}$$

From this relationship, we also see the following:

$$1 \text{ rad} = \left(\frac{180}{\pi}\right)^{\circ} \quad \text{and} \quad 1° = \frac{\pi}{180} \text{ rad}$$

Some examples

1) Change each radian measure to degree measure. Round to the nearest tenth of a degree, if necessary.

a) $\frac{\pi}{3}$ b) 2.2 c) -3π d) 1

2) Find the exact radian measure, in terms of π, for the following.

a) 45° b) 510° c) -210°

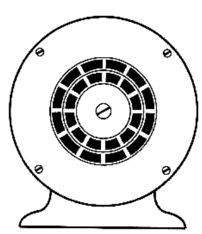
3) Change each degree measure to radian measure, to the nearest hundredth.

a) 30° b) -420°

4)

Angular Velocity

A small electric motor turns at 2000 r/min. Express the angular velocity, in radians per second, as an exact answer and as an approximate answer, to the nearest hundredth.



The abbreviation r means revolution.

SOLUTION

To complete one revolution, the motor turns through 2π radians.

$$2000 \text{ r/min} = 2000 \times 2\pi \text{ rad/min}$$

$$= \frac{2000 \times 2\pi}{60} \text{ rad/s}$$

$$\frac{2000 \times 2\pi}{60} = \frac{200\pi}{3}$$

$$\doteq 209.44$$

The angular velocity is exactly $\frac{200\pi}{3}$ rad/s, or 209.44 rad/s, to the nearest hundredth.