ROUND ALL ANSWERS TO THE NEAREST TENTH UNLESS STATED OTHERWISE

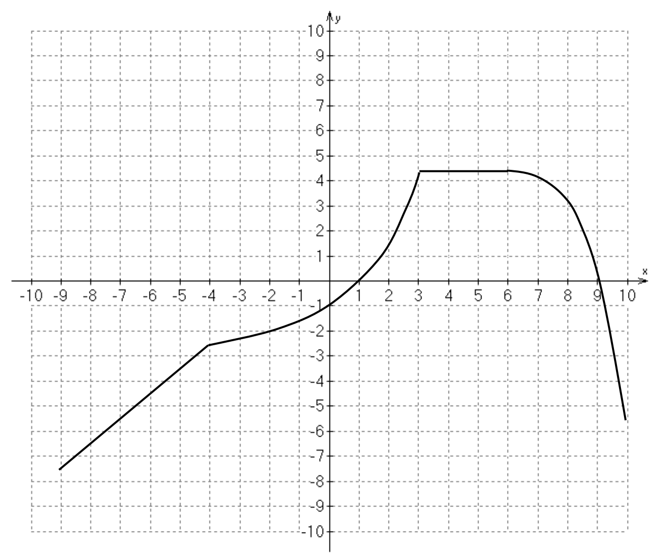
Multiple Choice

Identify the letter of the choice that best completes the statement or answers the question.

__a__ 1) The slope of a secant line on the graph of a function always gives
- a) an average rate of change
- b) an instantaneous rate of change
- c) the slope of a tangent line
- d) a non-zero value

__d__ 2) For the graph of a function, an instantaneous rate of change is always given by
- a) the midpoint of a secant line
- b) the y-value of a point of tangency
- c) the slope of a single secant line
- d) the slope of a single tangent line

Questions 3 through 5 refer to the graph of $y = f(x)$ shown on the right.



__b__ 3) At $x = 8$,
- a) the instantaneous rate of change of $f(x)$ is positive.
- b) the instantaneous rate of change of $f(x)$ is negative.
- c) the instantaneous rate of change of $f(x)$ is a maximum.
- d) the instantaneous rate of change of $f(x)$ is a minimum.

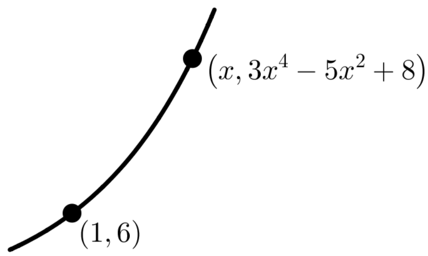
__c__ 4) On the interval $-9 < x < -5$,
- a) the rate of change of $f(x)$ is increasing.
- b) the average rate of change of $f(x)$ is negative.
- c) the average rate of change of $f(x)$ is equal to the instantaneous rate of change at any point.
- d) the instantaneous rate of change of $f(x)$ at any point is zero.

__c__ 5) On the interval $-4 < x < 3$,
- a) the rate of change of $f(x)$ is positive.
- b) the rate of change of $f(x)$ is increasing.
- c) both (a) and (b)
- d) neither (a) nor (b)

Full Solution

6) For the function $f(x)=3x^4-5x^2+8$, determine the **equation of the tangent line** at the point where $x = 1$.

$f(1)=6$



$$m_{\text{sec}}=\frac{3x^4-5x^2+8-6}{x-1}$$
$$=\frac{3x^4-5x^2+2}{x-1}$$

From left:

x	m_{sec}
0.9	0.817
0.99	1.871197
0.999	1.987011997
0.9999	1.99870012

From right:

x	m_{sec}
1.1	3.424
1.01	2.131203
1.001	2.013012003
1.0001	2.00130012

∴ the slope of the tangent is 2

Therefore,

$y=mx+b$

$y=2x+b$

$6=2(1)+b$

$b=4$

∴ the equation of the tangent line is $y=2x+4$

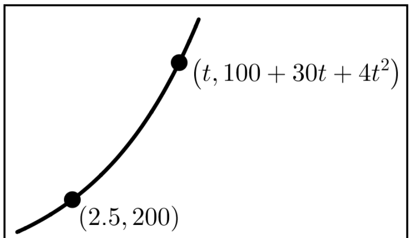
7) A population of raccoons moves into a wooded area. At t months, the number of raccoons, $P(t)$, can be modeled using the equation $P(t)=100+30t+4t^2$.

a) Determine the average rate of change of population over the first 5 months.

$$\text{A.R.O.C.}=\frac{P(5)-P(0)}{5-0}$$
$$=\frac{350-100}{5}$$
$$=\frac{250}{5}$$
$$= 50 \text{ raccoons/month}$$

b) Estimate the rate of change in the raccoon population at exactly 2.5 months. Round your final answer to the nearest whole number.

$P(2.5)=200$



$$m_{\text{sec}}=\frac{100+30t+4t^2-200}{t-2.5}$$
$$=\frac{4t^2+30t-100}{t-2.5}$$

From left:

t	m_{sec}
2.49	49.96
2.499	49.996
2.4999	49.9996
2.49999	49.99996

From right:

t	m_{sec}
2.51	50.04
2.501	50.004
2.5001	50.0004
2.50001	50.00004

∴ the rate of change is 50 raccoons/month

8) For the function $f(x)=2x^3+3x^2-72x+5$, use secant slopes to verify that a local maximum or minimum occurs at the point $(-4,213)$. State whether it is a maximum or a minimum that occurs at this point.

From left:

$$\begin{aligned} m_{\text{sec}} &= \frac{f(-4)-f(-4.001)}{-4-(-4.001)} \\ &= \frac{0.000021}{0.001} \\ &= 0.021 \end{aligned}$$

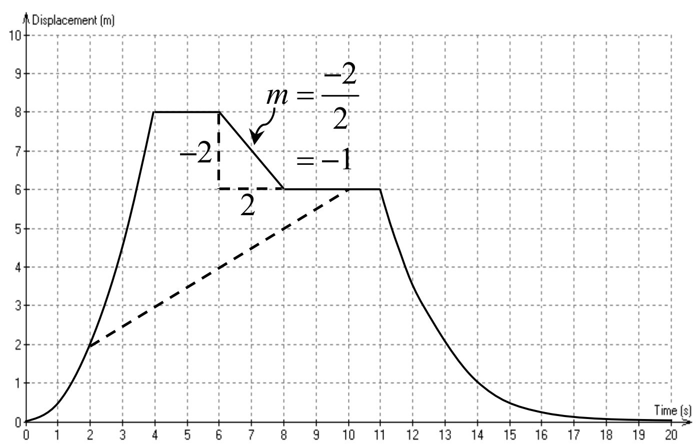
From right:

$$\begin{aligned} m_{\text{sec}} &= \frac{f(-3.999)-f(-4)}{-3.999-(-4)} \\ &= \frac{-0.000021}{0.001} \\ &= -0.021 \end{aligned}$$

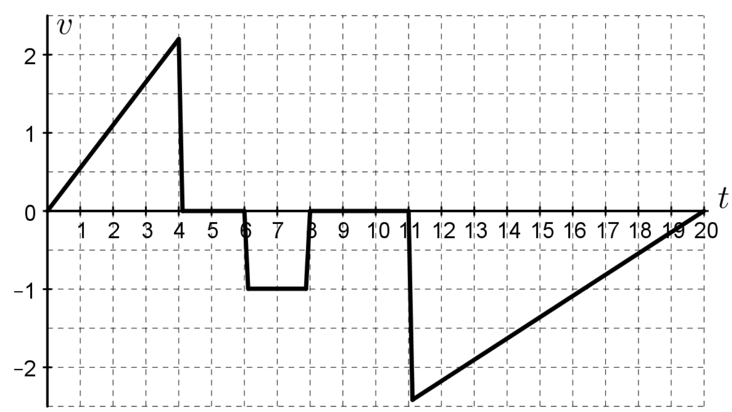
Based on the signs of the secant slopes above and their near-zero values, there is a local maximum at $(-4,213)$.

9) Consider the displacement-time graph shown below, where displacement is measured in metres and time is measured in seconds.

DISPLACEMENT-TIME GRAPH



VELOCITY-TIME GRAPH



a) Determine the average velocity in the interval from 2 seconds to 10 seconds.

$$\begin{aligned} \text{Avg. velocity} &= \frac{6-2}{10-2} \\ &= \frac{4}{8} \\ &= 0.5 \text{ m/s} \end{aligned}$$

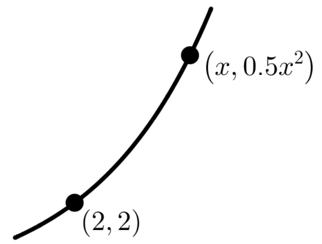
b) Determine the instantaneous velocity at 7 seconds.

Inst. velocity $= -1$ m/s (slope of line segment)

c) Draw a clearly labeled corresponding velocity-time graph on the provided axes.

d) For the displacement-time graph on the previous page, the first four seconds can be modeled by a quadratic function. Determine the instantaneous velocity at 2 seconds. (Hint: Look at the step pattern!)

Equation is $y = 0.5x^2$



$$m_{\sec} = \frac{0.5x^2 - 2}{x - 2}$$

From left:

x	$m_{\sec}$
1.9	1.95
1.99	1.995
1.999	1.9995
1.9999	1.99995

From right:

x	$m_{\sec}$
2.1	2.05
2.01	2.005
2.001	2.0005
2.0001	2.00005

∴ the instantaneous velocity is 2 m/s

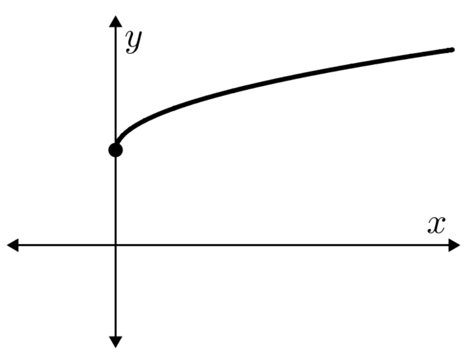
10) Determine the average rate of change of the function $f(x) = 2^x - \sqrt{x} + 5$ on the interval $2 \le x \le 7$. Round your **final answer** to the nearest tenth.

$$\text{A.R.O.C.} = \frac{f(7) - f(2)}{7 - 2}$$

$$= \frac{\left(2^7 - \sqrt{7} + 5\right) - \left(2^2 - \sqrt{2} + 5\right)}{5}$$

$$\approx 24.6$$

11) Elizabeth claims that she can find a point on the graph of $y = \sqrt{x} + 3$ at which the instantaneous rate of change is 0. Is Elizabeth's claim correct? Explain.



As shown on the left, the graph of $y = \sqrt{x} + 3$ never has a tangent slope of 0. The graph is always increasing, giving positive tangent slopes at all points (except at $x = 0$, where the tangent slope is undefined).

Therefore, the instantaneous rate of change is always positive (except at $x = 0$, where the tangent slope is undefined).

Therefore, Elizabeth's claim is incorrect.

12) For what values of x does the function $f(x) = 3(x+5)^2 + 7$ have

i) a positive instantaneous rate of change?

$x > -5$

ii) a negative instantaneous rate of change?

$x < -5$

iii) an instantaneous rate of change of 0?

$x = -5$