EXPRESS ALL FINAL ANSWERS IN EXACT FORM, UNLESS STATED OTHERWISE.

1) Solve the equation $x^4 + 2x^3 - 14x^2 = 13x - 30$. Round your final answer to the nearest tenth, if necessary.

$$x^4 + 2x^3 - 14x^2 - 13x + 30 = 0$$
$$(x+2)(x^3 - 14x + 15) = 0$$
$$(x+2)(x-3)(x^2 + 3x - 5) = 0$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)}$$
$$x \approx -4.2 \text{ or } x \approx 1.2$$

$\therefore x = -2, 3, -4.2 \text{ or } 1.2$

2) Solve the inequality $-2(x-4) \le 3x + 7$.

$$-2x + 8 \le 3x + 7$$
$$1 \le 5x$$
$$0.2 \le x$$

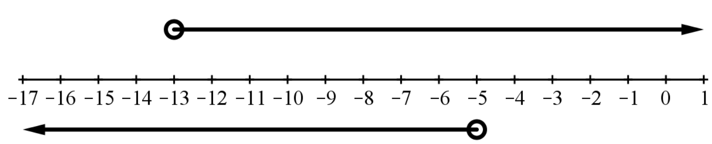
$\therefore x \ge 0.2$

3) Solve the double inequality $5x + 6 < 2x - 9 < 3x + 4$. If possible, express your final answer as a double inequality.

$5x + 6 < 2x - 9$ and $2x - 9 < 3x + 4$

$3x < -15$ $\quad$ $-x < 13$

$x < -5$ $\quad$ $x > -13$



$\therefore -13 < x < -5$

4) Solve the inequality $12x^4-41x^3+29x>36x^2-12$.

$$12x^4-41x^3-36x^2+29x+12>0$$
$$(x+1)(12x^3-53x^2+17x+12)>0$$
$$(x+1)(x-4)(12x^2-5x-3)>0$$
$$\underbrace{(x+1)(x-4)(3x+1)(4x-3)}_{f(x)}>0$$

Zeros are $-1,\ 4,\ -\frac{1}{3},\ \frac{3}{4}$

Interval	$x<-1$	$-1<x<-\frac{1}{3}$	$-\frac{1}{3}<x<\frac{3}{4}$	$\frac{3}{4}<x<4$	$x>4$
Sign of $f(x)$	+	-	+	-	+

$\therefore 12x^4-41x^3+29x>36x^2-12$ for $x<-1,\ -\frac{1}{3}<x<\frac{3}{4}$ or $x>4$

5) A new Toblerone chocolate bar box is to be designed to meet the following criteria:

- the box is to be a triangular prism with a face that is an isosceles triangle (see the diagram below)
- the height of the triangular face is 2 cm less than its base
- the length of the box is 9 cm greater than the base of the triangular face
- the volume of the box is 105 cm^3

Determine the dimensions (base, height and length) of the box. Your solution should also prove that there is only one box that meets the criteria. Label the dimensions on the diagram.

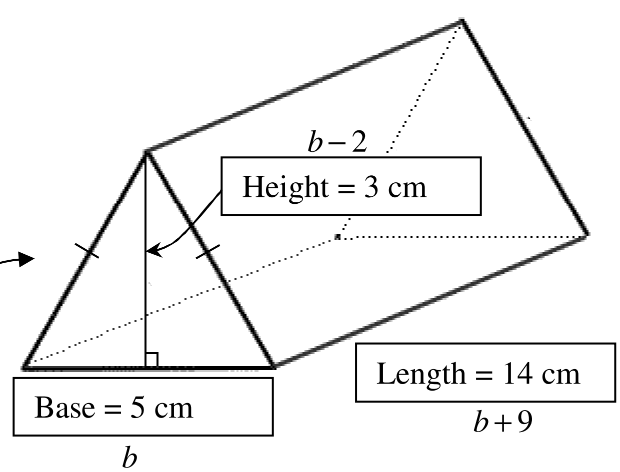
Note: The volume of a triangular prism is given by $V=\frac{1}{2}bhl$.

Let b repreent the length of the base.

$$V=\frac{1}{2}bhl$$
$$105=\frac{1}{2}b(b-2)(b+9)$$
$$210=b(b-2)(b+9)$$
$$210=b^3+7b^2-18b$$
$$0=b^3+7b^2-18b-210$$
$$0=(b-5)\underbrace{(b^2+12b+42)}_{\text{never zero (quadratic formula)}}$$

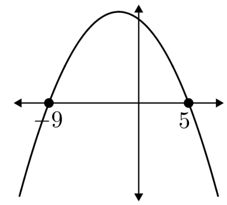
$\therefore b=5$ is the only solution

See diagram for dimensions.

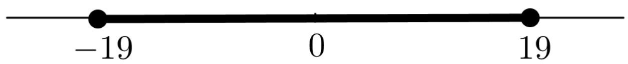


6) The inequality $a(x+b)(x+c)<0$ has the solution $x<-9$ or $x>5$. State a set of possible values for a, b and c.

$a =$ <u>-1</u> $\quad b =$ <u>9</u> $\quad c =$ <u>-5</u>

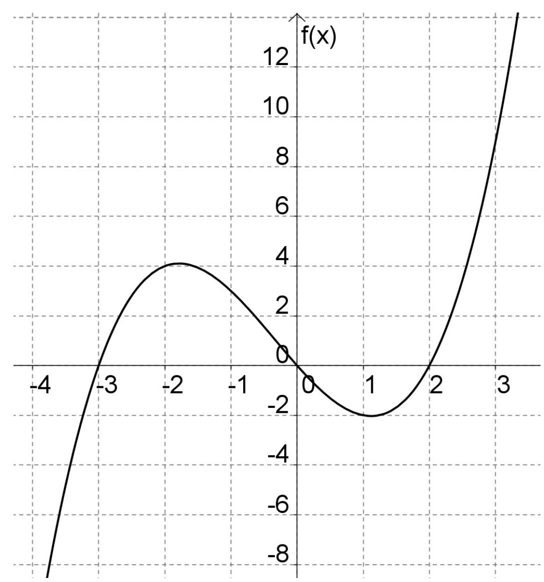


7) Solve the inequality $|2x+5| \le 19$.



$$\therefore\ -19 \le 2x+5 \le 19$$
$$-24 \le 2x \le 14$$
$$-12 \le x \le 7$$

8) Determine the instantaneous rate of change of the function shown in the graph on the right at the point where $x=-1$. Be sure to check the scales on the axes!



$f(x) = a(x+3)(x)(x-2)$

$(-2, 4)$ is on graph.

Therefore,

$4 = a(-2+3)(-2)(-2-2)$

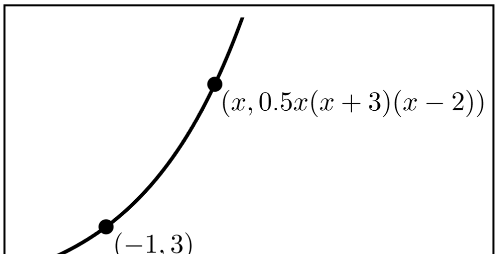
$4 = a(1)(-2)(-4)$

$4 = 8a$

$a = 0.5$

$\therefore\ f(x) = 0.5x(x+3)(x-2)$

$f(-1) = 3$



$$m_{sec} = \frac{\Delta f}{\Delta x} = \frac{0.5x(x+3)(x-2)-3}{x-(-1)} = \frac{0.5x(x+3)(x-2)-3}{x+1}$$

From left:

x	m_{sec}
−1.1	−2.395
−1.01	−2.48995
−1.001	−2.4989995
−1.0001	−2.4999

From right:

x	m_{sec}
−0.9	−2.595
−0.99	−2.50995
−0.999	−2.5009995
−0.9999	−2.5001

$\therefore$ the instantaneous rate of change at $x=-1$ is -2.5

9) Saul was solving a polynomial inequality when he accidentally spilled ketchup on his paper. The ketchup happened to land only on the exponents in his inequality, as shown below. If the given chart corresponds to Saul's inequality, fill in the correct numbers in the exponent positions. The function, $f(x)$, on the left side of the inequality is degree 7. **Be sure to place a number in every ketchup spot!**

$$-2(x+1)^{1}(x-5)^{2}(x+5)^{1}(2x-1)^{2}(3x-4)^{1} \geq 0$$

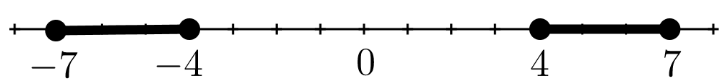
Interval	$x<-5$	$-5<x<-1$	$-1<x<\frac{1}{2}$	$\frac{1}{2}<x<\frac{4}{3}$	$\frac{4}{3}<x<5$	$x>5$
Sign of $f(x)$	+	-	+	+	-	-

10) Candice claims that in order to solve the inequality $4 \leq |2x+5| \leq 7$, we must have the following:

$$-4 \leq 2x+5 \leq -7 \qquad \text{or} \qquad 4 \leq 2x+5 \leq 7$$

Is Candice's claim correct? Explain.

Candice's claim is incorrect. $4 \leq |2x+5| \leq 7$ means that the distance of $2x+5$ from zero is between 4 and 7 (inclusive). So, $2x+5$ must lie between 4 and 7 (inclusive) or between -7 and -4 (inclusive).



$\therefore$ we must have $-7 \leq 2x+5 \leq -4$ or $4 \leq 2x+5 \leq 7$