ROUND ALL ANSWERS TO THE NEAREST TENTH, UNLESS STATED OTHERWISE

1) Evaluate each of the following. Only the final answer is required (you do not need to show any work). **All answers must be exact**.

a) $\log_7 49$

$= 2$

b) $\log_3 216 - \log_3 8$

$= \log_3 \frac{216}{8}$

$= \log_3 27$

$= 3$

c) $\log 100^9$

$= \log\left(10^2\right)^9$

$= 18$

d) $\log_k k^{90210}$

$= 90210$

e) $\log_{\frac{2}{3}}\left(\frac{16}{81}\right)$

$= 4$

f) $\log_2 \sqrt[3]{16}$

$= \log_2 16^{\frac{1}{3}}$

$= \log_2 \left(2^4\right)^{\frac{1}{3}}$

$= \log_2 2^{\frac{4}{3}}$

$= \frac{4}{3}$

2) Evaluate $\log_9 85$.

$$\log_9 85 = \frac{\log 85}{\log 9}$$

$$\approx 2.0$$

3) Solve the following equations.

a) $\log_5 x = 4$

$x = 5^4$

$x = 625$

b) $\log_9(4x+11) = \frac{1}{2}$

$9^{\frac{1}{2}} = 4x + 11$

$3 = 4x + 11$

$x = -2$

c) $9^{4x} = 27^{x+10}$

$\left(3^2\right)^{4x} = \left(3^3\right)^{x+10}$

$8x = 3x + 30$

$5x = 30$

$x = 6$

4) Solve the following equations.

a) $\log(2x+1) = \log(x-3) + \log 7$

$$\log(2x+1) = \log\left[7(x-3)\right]$$
$$2x+1 = 7(x-3)$$
$$2x+1 = 7x-21$$
$$22 = 5x$$
$$x \approx 4.4$$

b) $6(2)^{x+4} - 81 = 130$

$$6(2)^{x+4} = 211$$
$$2^{x+4} = \frac{211}{6}$$
$$x+4 = \log_2 \frac{211}{6}$$
$$x = \frac{\log \frac{211}{6}}{\log 2} - 4$$
$$x \approx 1.1$$

c) $\log_2(3x+1) + \log_2(x+8) = 3$

$$\log_2\left[(3x+1)(x+8)\right] = 3$$
$$3x^2 + 25x + 8 = 2^3$$
$$3x^2 + 25x = 0$$
$$x(3x+25) = 0$$
$$x = 0 \text{ or } \cancel{x = -\frac{25}{3}}$$

inadmissible

d) $5^{x+2} = 6^{x+1}$

$$\log 5^{x+2} = \log 6^{x+1}$$
$$(x+2)\log 5 = (x+1)\log 6$$
$$x\log 5 + 2\log 5 = x\log 6 + \log 6$$
$$x\log 5 - x\log 6 = \log 6 - 2\log 5$$
$$x(\log 5 - \log 6) = \log 6 - 2\log 5$$
$$x = \frac{\log 6 - 2\log 5}{\log 5 - \log 6}$$
$$x \approx 7.8$$

5) The sound level of a dog barking is 83 dB. The sound level of a thunderclap is 102 dB. How many times louder is the thunderclap than the dog?

$$102 - 83 = 19$$
$$L = 10\log \frac{I}{I_0}$$
$$19 = 10\log \frac{I}{I_0}$$
$$1.9 = \log \frac{I}{I_0}$$
$$10^{1.9} = \frac{I}{I_0}$$
$$\frac{I}{I_0} \approx 79.4$$

$\therefore$ the thunderclap is approximately 79.4 times louder than the dog barking

6) If it takes 251 days for 80 g of a radioactive substance to decay to 30.5 g, determine the half-life of the substance to the nearest tenth of a day.

$$30.5 = 80(0.5)^{\frac{251}{t}}$$

$$\frac{30.5}{80} = 0.5^{\frac{251}{t}}$$

$$\log\frac{30.5}{80} = \frac{251}{t}\log 0.5$$

$$t = \frac{251\log 0.5}{\log\frac{30.5}{80}}$$

$$t \approx 180.4 \text{ days}$$

7) Cassy Oh wants to be a millionaire. When she is 19 years old, she deposits \$3700 into an account that pays 5.75%/a compounded monthly. If Cassy does not make any more deposits or withdrawals in the account, **how old would she be** when the account reaches a value of \$1 000 000?

$$1000000 = 3700\left(1+\frac{0.0575}{12}\right)^{12t}$$

$$\frac{1000000}{3700} = \left(1+\frac{0.0575}{12}\right)^{12t}$$

$$\log\frac{1000000}{3700} = 12t\log\left(1+\frac{0.0575}{12}\right)$$

$$\frac{\log\frac{1000000}{3700}}{12\log\left(1+\frac{0.0575}{12}\right)} = t$$

$\therefore t \approx 97.6$ years

$\therefore$ she'll be 116

8) Determine the Richter scale measurement for an earthquake with approximately twice the intensity of tremor measuring 3.5 on the Richter scale?

$$10^x = 2\left(10^{3.5}\right)$$

$$x\log 10 = \log\left[2\left(10^{3.5}\right)\right]$$

$$x \approx 3.8$$

$\therefore$ Richter scale value is 3.8

9) Consider the function $f(x) = -2\log_3(x+4)$.

a) State the domain and range of $f(x)$.

D: $\{x \in \mathbb{R} \mid x > -4\}$

R: $\{y \in \mathbb{R}\}$

b) State the equation of the asymptote for the graph of $f(x)$.

$x = -4$

c) Determine the values of the function's x and y intercepts.

x-intercept:

$$0 = -2\log_3(x+4)$$
$$0 = \log_3(x+4)$$
$$3^0 = x+4$$
$$1 = x+4$$
$$x = -3$$

y-intercept:

$$f(0) = -2\log_3(0+4)$$
$$= -2\log_3 4$$
$$= -2\left(\frac{\log 4}{\log 3}\right)$$
$$\approx -2.5$$

d) Draw a sketch of the graph of $f(x)$.