NAME: SOLUTIONS

42

KNOW / 12	APP / 12	INQ / 12	COMM / 6

MCV4U1 - UNIT 7 – APPLICATIONS OF VECTORS
TEST

ROUND ALL FINAL ANSWERS TO THE NEAREST TENTH, UNLESS OTHERWISE STATED.

1) State whether the following expressions are vectors, scalars or meaningless (operations in brackets are completed first). (*K – 4 marks*)

a) $(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})$ vector

b) $\vec{a}\bullet(\vec{b}-\vec{c})$ scalar

c) $\vec{a}+(\vec{b}\bullet\vec{c})$ meaningless

d) $(\vec{a}\times\vec{b})\times(\vec{a}\bullet\vec{b})$ meaningless

2) For the vectors $\vec{a}=(3,-2,1)$ and $\vec{b}=(-4,5,3)$, determine each of the following.

a) $-2\vec{a}\bullet(3\vec{b}-\vec{a})$ (*K - 1 mark*)

$=(-6,4,-2)\bullet(-15,17,8)$

$=142$

b) $\vec{a}\times\vec{b}$ (*K - 1 mark*)

$=(-11,-13,-7)$

$\begin{matrix}3 & -2 & 1 & 3\\ -4 & 5 & 3 & -4\end{matrix}$

c) $\vec{a}\downarrow\vec{b}$ (*K - 1 mark*)

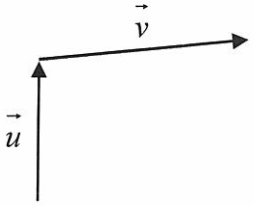
$=\dfrac{\vec{a}\bullet\vec{b}}{\vec{b}\bullet\vec{b}}\vec{b}$

$=\dfrac{-19}{50}(-4,5,3)$

$=\left(\dfrac{76}{50},-\dfrac{95}{50},\dfrac{-57}{50}\right)$

$=\left(\dfrac{38}{25},-\dfrac{19}{10},-\dfrac{57}{50}\right)$

3) For the diagram on the right, indicate whether $\vec{u} \times \vec{v}$ acts into or out of the page. (*K – 1 mark*)



Into the page.

4) Determine the angle, to the nearest tenth of a degree, between the vectors (4, 1, 2) and (3, -2, 5). (*K – 2 marks*)

$(4,1,2)\cdot(3,-2,5) = |(4,1,2)||(3,-2,5)|\cos\theta$

$20 = \sqrt{21}\sqrt{38}\cos\theta$

$\frac{20}{\sqrt{21}\sqrt{38}} = \cos\theta$

$\theta \doteq 44.9°$

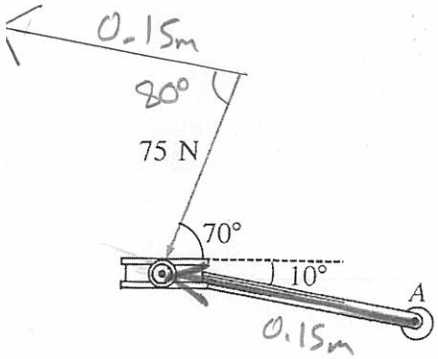
5) Determine the direction angles for the vector (2, -7, 3). (*K – 2 marks*)

$|(2,-7,3)| = \sqrt{62}$

$\cos\alpha = \frac{2}{\sqrt{62}}$ $\cos\beta = \frac{-7}{\sqrt{62}}$ $\cos\gamma = \frac{3}{\sqrt{62}}$

$\alpha \doteq 75.3°$ $\beta = 152.7°$ $\gamma = 67.6°$

6) A bicycle pedal is pushed by a 75 N force exerted by the foot of its rider, as shown in the following diagram. If the shaft of the pedal is 0.15 m long, find the magnitude of the torque about *A*. (*A – 3 marks*)



$|\vec{\tau}| = |\vec{r} \times \vec{F}|$

$= |\vec{r}||\vec{F}|\sin\theta$

$= (0.15)(75)\sin 80°$

$\doteq 11.1$ N·m

7) Determine if the vectors (1, 2, 3), (2, -1, 3) and (8, 1, 15) are coplanar. (*A – 2 marks*)

$(1,2,3) \times (2,-1,3) \cdot (8,1,15)$

$= (9, 3, -5)\cdot(8,1,15)$

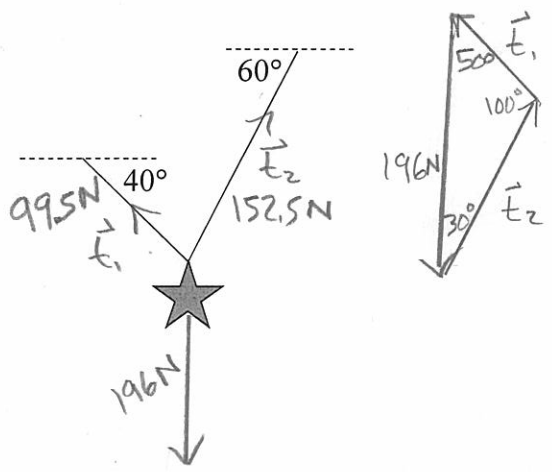
$= 72 + 3 - 75$

$= 0$

1 2 3 1
2 -1 3 2

∴ the vectors are coplanar.

8) An ornamental star that weighs 196 N is suspended by two chains fastened to horizontal beams at different heights, as shown in the diagram below. Determine the tension in each chain. (*A - 4 marks*)



$$\frac{|\vec{t}_1|}{\sin 30^\circ} = \frac{196}{\sin 100^\circ}$$

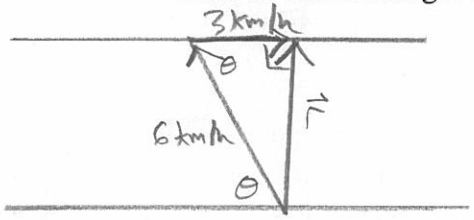
$$|\vec{t}_1| \doteq 99.5\text{ N}$$

$$\frac{|\vec{t}_2|}{\sin 50^\circ} = \frac{196}{\sin 100^\circ}$$

$$|\vec{t}_2| \doteq 152.5\text{ N}$$

∴ the tensions are 99.5 N and 152.5 N, as shown in the diagram.

9) a) A kayaker can paddle at a speed of 6 km/h in still water. If she wants to go straight across a river, and the current moves at 3 km/h, at what angle with the shore must she head in order to travel straight across the river. (*A - 3 marks*)



$$\cos\theta = \frac{3}{6}$$

$$\theta = 60^\circ$$

∴ she must head upstream at an angle of 60° to the shore.

10) Determine all **unit** vectors that are parallel to the *xy*-plane and perpendicular to the vector (1, -2, 2). Present your answer in exact form (you do not need to rationalize denominators). (*I – 5 marks*)

Vectors parallel to xy-plane have form $(x, y, 0)$

$\therefore (x, y, 0) \cdot (1, -2, 2) = 0$

$x - 2y = 0$

$x = 2y$

∴ need vector of form $(2y, y, 0)$

$|(2y, y, 0)| = \sqrt{(2y)^2 + y^2 + 0^2}$

$= \sqrt{5y^2}$

$= \sqrt{5}\,y$

→ unit vector is

$\frac{1}{\sqrt{5}y}(2y, y, 0)$

$= \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right)$

∴ the possible vectors are $\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right)$ and $\left(-\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}, 0\right)$

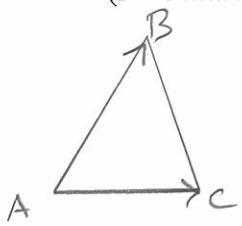
11) If $|\vec{a}| = 4$ and $|\vec{b}| = 7$ ~~all unit vectors~~ and $|\vec{a} \downarrow \vec{b}| = \sqrt{3}$, determine the exact value of $\vec{a} \bullet \vec{b}$.
(*I – 3 marks*)

$|\vec{a} \downarrow \vec{b}| = \dfrac{\vec{a} \bullet \vec{b}}{|\vec{b}|}$

$\therefore \sqrt{3} = \dfrac{\vec{a} \bullet \vec{b}}{7}$

$\therefore \vec{a} \bullet \vec{b} = 7\sqrt{3}$

12) Triangle ABC has vertices $A(2, 1, 3)$, $B(4, 1, 5)$ and $C(3, 2, 6)$. Find the area of the triangle.
(*I – 4 marks*)



$\vec{AB} = (2, 0, 2)$

$\vec{AC} = (1, 1, 3)$

$2 \quad 0 \quad 2 \quad 2$
$1 \quad 1 \quad 3 \quad 1$

$\text{Area} = |\vec{AB} \times \vec{AC}| \div 2$

$= |(-2, -4, 2)| \div 2$

$= \sqrt{24} \div 2$

$= \sqrt{6}$

$\doteq 2.4 \text{ units}^2$

13) If $\vec{a} \bullet \vec{b} = \vec{a} \bullet \vec{c}$, is it always true that $\vec{b} = \vec{c}$? Explain. (*C – 3 marks*)

No. For example, if $\vec{a} = (1,1,1)$, $\vec{b} = (1,2,1)$ and $\vec{c} = (1,0,3)$, then $\vec{a} \bullet \vec{b} = 4$ and $\vec{a} \bullet \vec{c} = 4$, but $\vec{b} \neq \vec{c}$.

14) Prove that the dot product is distributive over addition in 3-space. That is, prove that $\vec{u} \bullet (\vec{v} + \vec{w}) = \vec{u} \bullet \vec{v} + \vec{u} \bullet \vec{w}$ in $\Re^3$. (*C – 3 marks*)

Let $\vec{u} = (a, b, c)$, $\vec{v} = (d, e, f)$ and $\vec{w} = (g, h, i)$

L.S.

$\vec{u} \bullet (\vec{v} + \vec{w})$

$= (a, b, c) \bullet [(d, e, f) + (g, h, i)]$

$= (a, b, c) \bullet (d+g, e+h, f+i)$

$= a(d+g) + b(e+h) + c(f+i)$

$= ad + ag + be + bh + cf + ci$

R.S.

$\vec{u} \bullet \vec{v} + \vec{u} \bullet \vec{w}$

$= (a, b, c) \bullet (d, e, f) + (a, b, c) \bullet (g, h, i)$

$= ad + be + cf + ag + bh + ci$

$= ad + ag + cf + bh + cf + ci$

$\therefore$ L.S. = R.S

$\therefore \vec{u} \bullet (\vec{v} + \vec{w}) = \vec{u} \bullet \vec{v} + \vec{u} \bullet \vec{w}$