THE DOT PRODUCT OF ALGEBRAIC VECTORS

Using our knowledge of operations on Cartesian vectors, we can derive the following property for the dot product of Cartesian vectors.

The Dot Product of Cartesian Vectors

If $\vec{a} = (a_1, a_2)$ and $\vec{b} = (b_1, b_2)$, then

$$\vec{a} \bullet \vec{b} = a_1 b_1 + a_2 b_2$$

Therefore, to determine the dot product of vectors in Cartesian form, add the products of corresponding components.

Example

Find the dot product of $\vec{u}$ and $\vec{v}$, where $\vec{u} = (2, -4)$ and $\vec{v} = (3, 5)$.

Using the Dot Product to Find the Angle Between Two Vectors

By solving the dot product definition $\vec{u} \bullet \vec{v} = \left|\vec{u}\right|\left|\vec{v}\right|\cos\theta$ for $\cos\theta$, we obtain a useful formula for calculating the angle between two vectors.

$$\cos\theta = \frac{\vec{u} \bullet \vec{v}}{\left|\vec{u}\right|\left|\vec{v}\right|}$$

NOTE: Instead of memorizing this formula, just remember how it is obtained from the definition of the dot product.

Example

Find the angle between $\vec{u} = (1, -3, 2)$ and $\vec{v} = (2, 7, -1)$.